Riemann Sums

- RRAM (Right Rectangular Approximation Method)
- LRAM (Left Rectangular Approximation Method)
- MRAM (Midpoint Rectangular Approximation Method)

1. A particle starts at $x = 0$ and moves along the x-axis with velocity $v(t) = t^2 + 2$ for time $t \geq 0$. Where is the particle at $t = 5$? Approximate the area under the curve using five rectangles of equal width and heights determined by the midpoints of the intervals.

2. Use RRAM with $n = 5$ to estimate the area of the region enclosed between the graph of f and the x-axis for $a \leq x \leq b$

   $f(x) = \sin x$, $a = 0$, $b = \pi$

3.

Distance Traveled The table below shows the velocity of a model train engine moving along a track for 10 sec. Estimate the distance traveled by the engine, using 10 subintervals of length 1 with **(a)** left-endpoint values (LRAM) and **(b)** right-endpoint values (RRAM).

Time (sec)	Velocity (in./sec)	Time (sec)	Velocity (in./sec)
0	0	6	11
1	12	7	6
2	22	8	2
3	10	9	6
4	5	10	0
5	13		

THEOREM 1 The Existence of Definite Integrals

All continuous functions are integrable. That is, if a function f is continuous on an interval $[a, b]$, then its definite integral over $[a, b]$ exists.

The Definite Integral of a Continuous Function on $[a, b]$

Let f be continuous on $[a, b]$, and let $[a, b]$ be partitioned into n subintervals of equal length $\Delta x = (b - a)/n$. Then the definite integral of f over $[a, b]$ is given by

$$\lim_{n \to \infty} \sum_{k=1}^{n} f(c_k) \Delta x,$$

where each c_k is chosen arbitrarily in the k^{th} subinterval.

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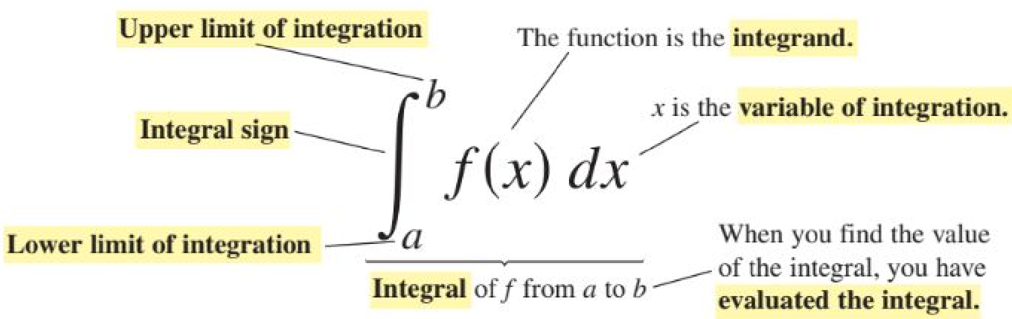
$$\lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\Delta x = \int_a^b f(x)\,dx.$$



DEFINITION Area Under a Curve (as a Definite Integral)

If $y = f(x)$ is nonnegative and integrable over a closed interval $[a, b]$, then the **area under the curve $y = f(x)$ from a to b** is the integral of f from a to b,

$$A = \int_a^b f(x)\,dx.$$

$$\text{Area} = -\int_a^b f(x)\,dx \quad \text{when} \quad f(x) \le 0.$$

$$\int_a^b f(x)\,dx = (\text{area above the } x\text{-axis}) - (\text{area below the } x\text{-axis}).$$

THEOREM 2 The Integral of a Constant

If $f(x) = c$, where c is a constant, on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = \int_a^b c\,dx = c(b - a).$$

5. Use the graph of the integrand and area to evaluate the integral:

$$\int_{-2}^{2} (2 - |x|)\, dx$$

6. Use areas to evaluate the integral:

   a. $\int_{0}^{b} 4x\, dx, \quad b > 0$

   b. $\int_{a}^{\sqrt{3}a} x\, dx, \quad a > 0$

7. Find the points of discontinuity of the integrand on the interval of integration, and use area to evaluate the integral.

$$\int_{-5}^{6} \frac{9-x^2}{x-3}\, dx$$

Table 5.3 Rules for Definite Integrals

1. *Order of Integration:* $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ — A definition

2. *Zero:* $\int_a^a f(x)\,dx = 0$ — Also a definition

3. *Constant Multiple:* $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ — Any number k

   $\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$ — $k = -1$

4. *Sum and Difference:* $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. *Additivity:* $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

6. *Max-Min Inequality:* If max f and min f are the maximum and minimum values of f on $[a, b]$, then

$$\min f \cdot (b - a) \le \int_a^b f(x)\,dx \le \max f \cdot (b - a).$$

7. *Domination:* $f(x) \ge g(x)$ on $[a, b] \Rightarrow \int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

   $f(x) \ge 0$ on $[a, b] \Rightarrow \int_a^b f(x)\,dx \ge 0$ — $g = 0$

9. Suppose that f and g are continuous functions and that

$$\int_1^2 f(x)dx = -3, \quad \int_1^5 f(x)dx = 8, \quad \int_1^5 g(x)dx = 10$$

Find each integral below:

a. $\int_2^2 g(x)dx$

b. $\int_5^1 g(x)dx$

c. $\int_1^2 3f(x)dx$

d. $\int_2^5 f(x)dx$

e. $\int_1^5 [f(x) - g(x)]dx$

f. $\int_1^5 [4f(x) - g(x)]dx$

10. Suppose that h is continuous and that

$$\int_{-1}^{1} h(r)dr = 0 \qquad \text{and} \qquad \int_{-1}^{3} h(r)dr = 5$$

Find each integral.

a. $\int_{1}^{3} h(r)dr$

b. $-\int_{3}^{1} h(u)du$

11. Interpret the integrand as the rate of change of a quantity and evaluate the integral using the antiderivative of the quantity:

a. $\int_{0}^{\frac{\pi}{2}} \cos x\, dx$

b. $\int_{0}^{\frac{\pi}{4}} \sec^2 x\, dx$

c. $\int_{0}^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx$

DEFINITION Average (Mean) Value

If f is integrable on $[a, b]$, its **average (mean) value** on $[a, b]$ is

$$av(f) = \frac{1}{b-a}\int_{a}^{b} f(x)\, dx.$$

12. Find the average value of the function on the interval, using antiderivatives to compute the integral.

a. $y = \frac{1}{x}, \quad [e, 2e]$

b. $y = \frac{1}{1+x^2}, \quad [0, 1]$

c. $y = \sec x \tan x, \quad [0, \frac{\pi}{3}]$

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Most Difficult First:

Pg. 278: #19
Pg. 292: #52, 54
Pg. 300: #40