Geometry CP
6.1 Angles of Polygons

Polygon: a closed figure formed by a finite # of coplanar segments that have a common endpoint are noncollinear and each side intersects exactly 2 other side at the endpoints

ex) 

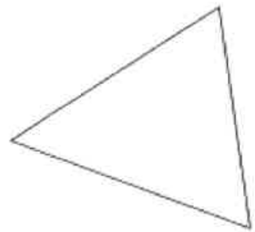
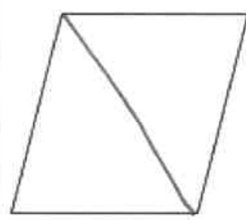
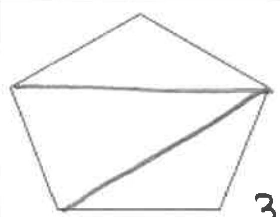
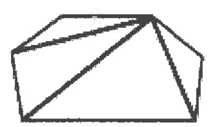
non ex) 

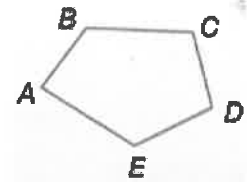
Diagonal: a segment that connects any 2 nonconsecutive vertices

ex)  diagonal

Using 1 vertex construct all diagonals → creates triangles inside polygon

Investigate!

Polygon	Number of Sides	Number of Triangles (construct all possible diagonals from one vertex)	Sum of Interior Angle Measure
Triangle	3	 1	180
Quadrilateral	4	 2	2(180) = 360
Pentagon	5	 3	3(180) = 540
Hexagon	6	 4 Triangles	4(180) = 720
n-gon	n	(n - 2)	(n-2) 180

Polygon Interior Angle Sum Theorem	The sum of the interior angle measure of an n – sided convex polygon is $(n-2)\cdot 180$	

Recall:

Number of Sides	Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octogon
9	Nonagon
10	Decagon
11	Hendecagon
12	Dodecagon
n	n-gon

equilateral polygon : polygon w/ all congruent sides

equiangular polygon: polygon w/ all congruent angles

regular polygon: convex polygon that is both equilateral and equiangular

convex: all diagonals lie inside the polygon

concave is not convex

ex)

concave

1. Find the sum of the measure of the interior angles of a convex heptagon.

$(7-2)180 = 900^\circ$

7 sides

2. Find the measure of ONE angle in each regular polygon below:

all angles equal

a. Regular 18-gon

$(18-2)180 = 2880^\circ$

2880 = sum of all 18 angles

$\frac{2880}{18} = \boxed{160^\circ}$

b. Regular 24-gon

$\frac{(24-2)180}{24} = \boxed{165^\circ}$

c. Regular 15-gon

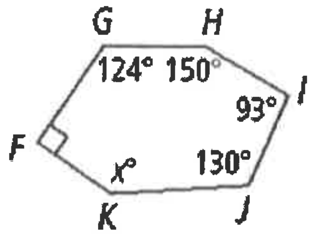
$\frac{(15-2)180}{15} = \boxed{156^\circ}$

3. Find the measure of each interior angle of:

a.

6 sides

$(6-2)180 = 720$

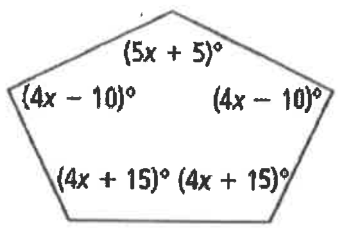


$720 = 90 + 124 + 150 + 93 + 130 + x$

$720 = 587 + x$

$\boxed{133° = x}$

b.



5 sides

$(5-2)180 = 540$

$540 = 5x+5 + 4x-10 + 4x+15$
$+ 4x + 15 + 4x - 10$

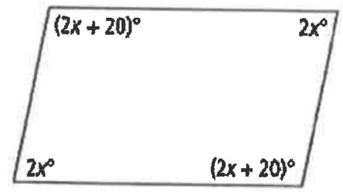
$540 = 21x + 15$

$525 = 21x$

$\boxed{25° = x}$

c.

4 sides → 360°



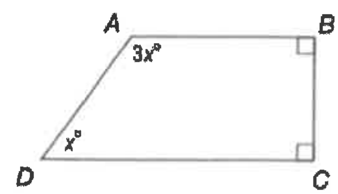
$360 = 2x + 2x + 20 + 2x +$
$2x + 20$

$360 = 8x + 40$

$320 = 8x$

$\boxed{40° = x}$

d.



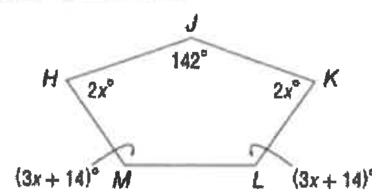
$360 = 90 + 90 + x + 3x$

$360 = 180 + 4x$

$180 = 4x$

$\boxed{45° = x}$

e.



$540 = 2x + 142 + 2x + 3x + 14 +$
$3x + 14$

$540 = 10x + 170$

$370 = 10x$

$\boxed{37° = x}$

4. The measure of an interior angle of a regular polygon is 135. Find the number of sides in the polygon.

$135 = \frac{(n-2)180}{n}$

$135n = (n-2)180$

$135n = 180n - 360$

$-45n = -360$

$n = 8$

$\boxed{8 \text{ sides}}$

5. The measure of an interior angle of a regular polygon is 144. Find the number of sides in the polygon.

$144 = \frac{(n-2)180}{n}$

$144n = 180n - 360$

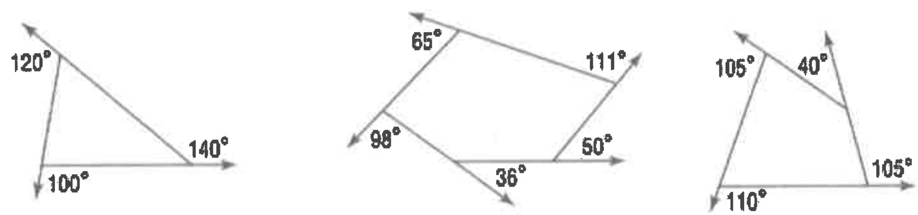
$-36n = -360$

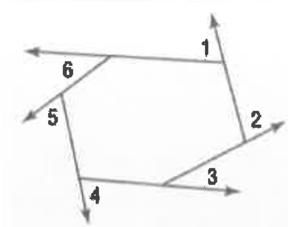
$n = 10$

$\boxed{10 \text{ sides}}$

Investigate!

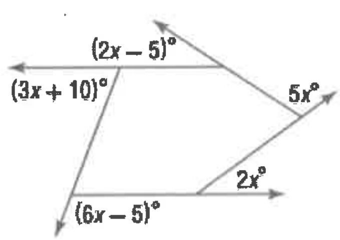
Does a relationship exist between the number of sides of a convex polygon and the sum of its exterior angle measures?



Polygon Exterior Angle Sum Theorem	The sum of the exterior angle measure of a convex polygon, one angle at each vertex, is 360.	

6. Find the value of x in the diagram:

a.

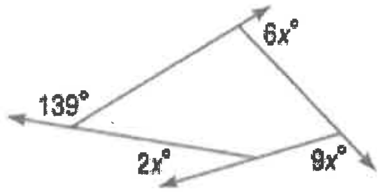


$360 = 2x - 5 + 5x + 2x + 6x - 5 + 3x + 10$

$360 = 18x$

$\boxed{20^{\circ} = x}$

b.



$360 = 6x + 9x + 2x + 139$

$360 = 17x + 139$

$221 = 17x$

$\boxed{13° = x}$

7. Find the measure of an exterior angle of each regular polygon:
   a. 80-gon
   b. 20-gon
   c. 19-gon

a. sum of exterior = 360

80 sides = 80 exterior angles

$\frac{360}{80} = \frac{9}{2} = \boxed{4.5°}$

b. $\frac{360}{20} = \boxed{18°}$

c. $\frac{360}{19} \approx \boxed{18.95°}$

Summary

sum of interior angles = $(n-2)180$

one interior angle = $\frac{(n-2)180}{n}$
* regular

sum of exterior angles = 360°

one exterior angle = $\frac{360}{n}$
* regular