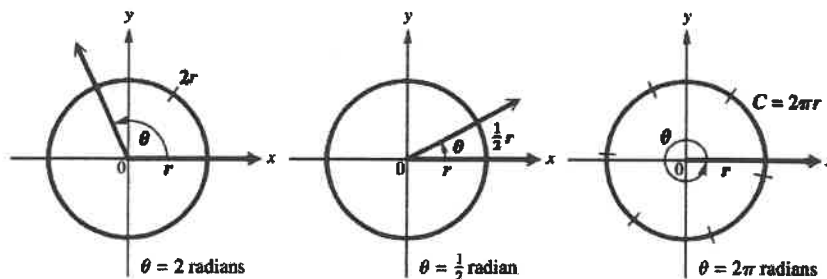


Radian Measure

Radian

An angle with its vertex at the center of a circle that intercepts an arc on the circle equal in length to the radius of the circle has a measure of 1 radian.



In general, if θ is a central angle of radius r , and θ intercepts an arc of length s , then the radian measure of θ is $\frac{s}{r}$.

The ratio $\frac{s}{r}$ is a pure number, where s and r are expressed in the same units. Thus, "radians" is not a unit of measure like feet or centimeters.

Conversions between Degrees and Radians

$$360^\circ = \frac{2\pi}{1} \text{ radians}$$

$$180^\circ = \frac{1}{2} (2\pi) \text{ radians} = \pi \text{ radians}$$

$$1^\circ = \frac{\pi}{180} \text{ radian} \quad \text{or} \quad 1 \text{ radian} = \frac{180^\circ}{\pi}^\circ$$

Converting between Degrees and Radians

1. Multiply a degree measure by $\frac{\pi}{180}$ radian and simplify to convert to radians.
2. Multiply a radian measure by $\frac{180^\circ}{\pi}$ and simplify to convert to degrees.

1. Convert each degree measure to radians.

a. 108°

$$\begin{aligned} 108 \cdot \frac{\pi}{180} \\ = \frac{3\pi}{5} \end{aligned}$$

b. -135°

$$\begin{aligned} -135 \cdot \frac{\pi}{180} \\ = -\frac{3\pi}{4} \end{aligned}$$

c. 325.7°

$$\begin{aligned} 325.7 \cdot \frac{\pi}{180} \\ = 5.685 \end{aligned}$$

6.1 Radian Measure
Honors Algebra 2 with Trig

2. Convert each radian measure to degrees.

$$(a) \frac{11\pi}{12} \cdot \frac{180}{\pi} = 165^\circ$$

$$(b) -\frac{7\pi}{6} \cdot \frac{180}{\pi} = -210^\circ$$

$$(c) -2.92 \cdot \frac{180}{\pi} = -167.3^\circ$$

Agreement on Angle Measurement Units

If no unit of angle measure is specified, then the angle is understood to be measured in radians.

For example, the figure on the left shows an angle of 30° , and the figure on the right shows an angle of 30 (which means 30 radians). An angle with measure 30 radians is coterminal with an angle of approximately 279° .

(a) 30 degrees

(b) 30 radians

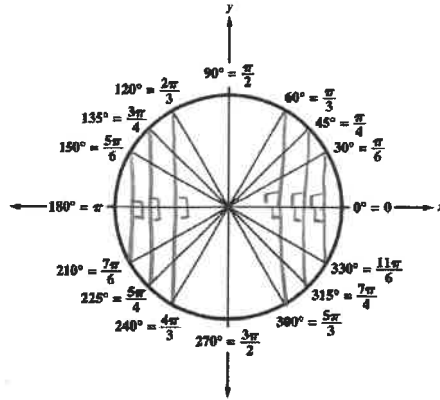
Note the difference between an angle of 30 degrees and an angle of 30 radians.

Equivalent Angle Measures

Degrees	Radians		Degrees	Radians	
	Exact	Approximate		Exact	Approximate
0°	0	0	90°	$\frac{\pi}{2}$	1.57
30°	$\frac{\pi}{6}$	0.52	180°	π	3.14
45°	$\frac{\pi}{4}$	0.785	270°	$\frac{3\pi}{2}$	4.71
60°	$\frac{\pi}{3}$	1.047	360°	2π	6.28

These exact values are rational multiples of π .

6.1 Radian Measure
Honors Algebra 2 with Trig



Learn the equivalences in this figure. They will appear often in trigonometry.

Arc Length on a Circle

Arc Length

The length s of the arc intercepted on a circle of radius r by a central angle of measure θ radians is given by the product of the radius and the radian measure of the angle.

$s = r\theta$, where θ is in radians.

When the formula $s = r\theta$ is applied, the value of θ MUST be expressed in radians, not degrees.

★ $\frac{\text{Arc}}{2\pi r} = \frac{\theta}{360^\circ}$ ★

Arc = $\frac{\theta \cdot 2\pi r}{360^\circ}$

If cancel 2π & 360° then θ must be in radians!

3. A circle has radius 25.60 cm. Find the length of the arc intercepted by a central angle having each of the following measures.

$r = 25.60$

a. $\frac{7\pi}{8}$

$s = r\theta$

$s = 25.6 \left(\frac{7\pi}{8}\right)$

$= 70.37 \text{ cm}$

b. 54°

$\frac{s}{2\pi r} = \frac{\theta}{360^\circ}$

★ or convert to radians & use $s = r\theta$

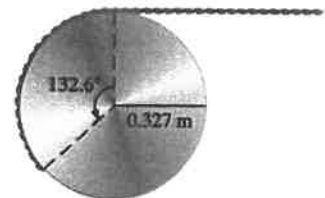
$s = \frac{54^\circ}{360^\circ} \cdot 2\pi (25.60)$

$\frac{2\pi}{360} = \frac{\pi}{180}$
so converting in problem

$= 24.13 \text{ cm}$

- 4.

A rope is being wound around a drum with radius 0.327 m. (See the figure.) How much rope will be wound around the drum if the drum is rotated through an angle of 132.6° ?



$\theta = 132.6^\circ$ $r = 0.327 \text{ m}$

$\frac{s}{2\pi(0.327)} = \frac{132.6^\circ}{360^\circ}$

$s = \frac{132.6^\circ}{360^\circ} \cdot 2\pi(0.327)$

$= 0.757 \text{ m}$

Area of a Sector of a Circle



A sector of a circle is the portion of the interior of a circle intercepted by a central angle.

Area of a Sector
The area A of a sector of a circle of radius r and central angle θ is given by the following formula.

$$A = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

$$\frac{A}{\pi r^2} = \frac{\theta}{360^\circ}$$

As in the formula for arc length, the value of θ must be in radians when this formula is used for the area of a sector.

5. Find the area of a sector of a circle having radius 15.20 ft and central angle 108.0° .

$$\frac{A}{\pi r^2} = \frac{\theta}{360^\circ}$$
$$\frac{A}{\pi (15.2)^2} = \frac{108^\circ}{360^\circ}$$
$$A = \frac{108^\circ}{360^\circ} \pi (15.2)^2$$
$$= 217.75 \text{ ft}^2$$