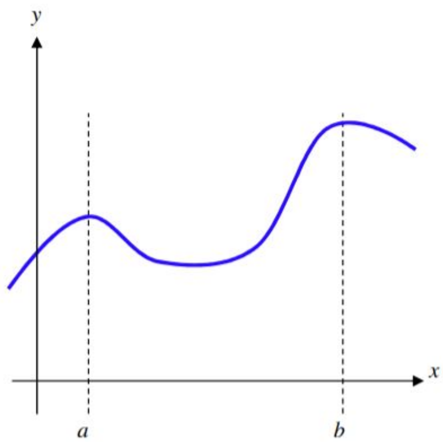
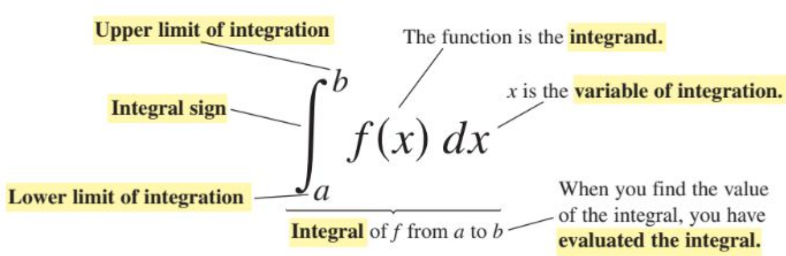


Width of each rectangle = ______________

*each rectangle does not need to have the same width, but we will assume it for this



Read as "the integral of f of x from a to b"

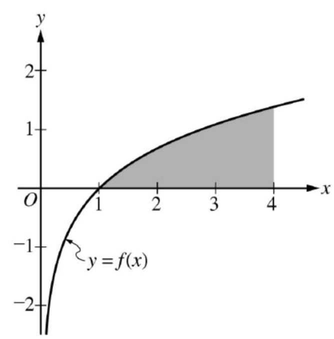
Important: If the function is CONTINUOUS then the definite integral WILL exist.
The reverse is true sometimes, but not always.

5. The interval $[-1,\ 3]$ is partitioned into n subintervals of equal length $\Delta x = \frac{4}{n}$. Let m_k denote the midpoint of the k^{th} subinterval. Express the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} (3(m_k)^2 - 2m_k + 5)\Delta x$$

as an integral.

5. The function f is given by $f(x) = \ln x$. The graph of f is shown below. Which of the following is equal to the area of the shaded region?



(A) $\lim\limits_{x\to\infty} \sum\limits_{k=1}^{n} \left(1 + \ln\left(\frac{3k}{n}\right)\right) \frac{3}{n}$

(B) $\lim\limits_{x\to\infty} \sum\limits_{k=1}^{n} \ln\left(1 + \left(\frac{3k}{n}\right)\right) \frac{3}{n}$

(C) $\lim\limits_{x\to\infty} \sum\limits_{k=1}^{n} \ln\left(\frac{4}{n}\right)\left(1 + \frac{4k}{n}\right)$

(D) $\lim\limits_{x\to\infty} \sum\limits_{k=1}^{n} \ln\left(1 + \frac{4k}{n}\right)\left(\frac{4}{n}\right)$

6. The closed interval $[a, b]$ is partitioned into n equal sub intervals each of width Δx, by the numbers $x_0,\ x_1,\ \ldots,\ x_n$ where $a = x_0 < x_1 < \ldots < x_{n-1} < x_n = b$. Express $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \sqrt{x_i}\,\Delta x$ as an integral.

7. For a certain continuous function, f, the right Riemann sum approximation of $\int_0^2 f(x)dx$ with n subintervals of equal length is $\frac{2(n+1)(3n+2)}{n^2}$ for all n. What is the value of $\int_0^2 f(x)dx$?

A) 2 C) 12

B) 6 D) 20

8. Which of the following limits is equal to $\int_2^5 x^2 dx$

A) $\lim_{x\to\infty} \sum_{k=1}^{n} \left(2+\frac{k}{n}\right)^2 \frac{1}{n}$

C) $\lim_{x\to\infty} \sum_{k=1}^{n} \left(2+\frac{3k}{n}\right)^2 \frac{1}{n}$

B) $\lim_{x\to\infty} \sum_{k=1}^{n} \left(2+\frac{k}{n}\right)^2 \frac{3}{n}$

D) $\lim_{x\to\infty} \sum_{k=1}^{n} \left(2+\frac{3k}{n}\right)^2 \frac{3}{n}$

9. The continuous function f is decreasing for all x. Selected values of f are given in the table below, where a is a constant with $0 < a < 3$. Let R be the right Riemann sum approximation for $\int_0^{7a} f(x)dx$ using four subintervals indicated by the data in the table. Which of the following statements is true?

x	0	a^2	$3a$	$6a$	$7a$
$f(x)$	1	-1	-3	-7	-9

A) $R=(a^2-0)\cdot 1+(3a-a^2)\cdot(-1)+(6a-3a)\cdot(-3)+(7a-6a)\cdot(-7)$ and is an underestimate for $\int_0^{7a} f(x)\,dx$.

B) $R=(a^2-0)\cdot 1+(3a-a^2)\cdot(-1)+(6a-3a)\cdot(-3)+(7a-6a)\cdot(-7)$ and is an overestimate for $\int_0^{7a} f(x)\,dx$.

C) $R=(a^2-0)\cdot(-1)+(3a-a^2)\cdot(-3)+(6a-3a)\cdot(-7)+(7a-6a)\cdot(-9)$ and is an underestimate for $\int_0^{7a} f(x)\,dx$.

D) $R=(a^2-0)\cdot(-1)+(3a-a^2)\cdot(-3)+(6a-3a)\cdot(-7)+(7a-6a)\cdot(-9)$ and is an overestimate for $\int_0^{7a} f(x)\,dx$.

10. Which of the following is a left Riemann sum approximation of $\int_2^8 \cos(x^2)dx$ with n subintervals of equal length?

(A) $\sum_{k=1}^{n}\left(\cos\left(2+\frac{k-1}{n}\right)^{2}\right)\frac{1}{n}$

(B) $\sum_{k=1}^{n}\left(\cos\left(\frac{6k}{n}\right)^{2}\right)\frac{6}{n}$

(C) $\sum_{k=1}^{n}\left(\cos\left(2+\frac{6(k-1)}{n}\right)^{2}\right)\frac{6}{n}$

(D) $\sum_{k=1}^{n}\left(\cos\left(2+\frac{6k}{n}\right)^{2}\right)\frac{6}{n}$

Area Under a Curve

If $y=f(x)$ is nonnegative and integrable over a closed interval $[a,b]$, then the area under the curve $y=f(x)$ from a to b is the integral of f from a to b,

$A = \int_a^b f(x)dx$	
When $f(x) \le 0$ Area = ________________	
$\int_a^b f(x)dx =$ ____	

11. Evaluate the integral below:

    a. $\int_3^7 (-20)\,dx$

12. Use the graph of the integrand and area to evaluate the integral:

    a. $\int_{-1}^{1} (1-|x|)\,dx$

    b. $\int_{\frac{1}{2}}^{\frac{3}{2}} (-2x+4)\,dx$

    c. $\int_{-4}^{0} \sqrt{16-x^2}\,dx$

13. Use areas to evaluate the integral:

a. $\int_0^b 4x\,dx, \quad b > 0$

b. $\int_a^{\sqrt{3}a} x\,dx, \quad a > 0$

14. Find the points of discontinuity of the integrand on the interval of integration, and use area to evaluate the integral.

a. $\int_{-5}^{6} \frac{9-x^2}{x-3}\,dx$