

Width of each rectangle =  $\Delta x$

\*each rectangle does not need to have the same width, but we will assume if for this

$$A = \text{Rectangle}_1 + \text{Rectangle}_2 + \dots + \text{Rectangle}_n$$

$$\text{Area Rectangle} = (\text{width})(\text{height})$$

$\downarrow$   
function value

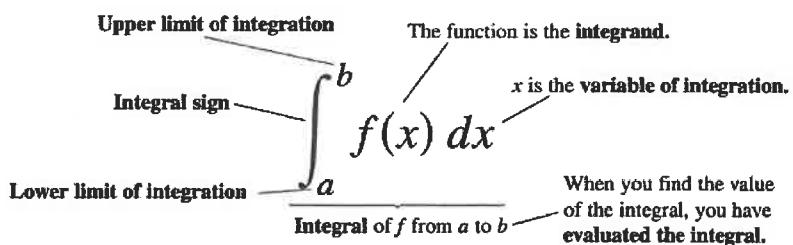
$$A = \Delta x f(c_1) + \Delta x f(c_2) + \Delta x f(c_3) + \dots + \Delta x f(c_n)$$

$$A = \sum_{k=1}^n \Delta x f(c_k)$$

$$A = \sum_{k=1}^n f(c_k) \Delta x$$

$$A = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(c_k) \Delta x$$

\*Definite Integral = limit of Riemann Sums  
send the # of rectangles to  $\infty$   
to get exact area



Read as "the integral of  $f$  of  $x$  from  $a$  to  $b$ "

Important: If the function is CONTINUOUS then the definite integral WILL exist.

The reverse is true sometimes, but not always.

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n f(c_k) \Delta x = \int_a^b f(x) dx$$

$$\frac{b-a}{n} = \frac{3-(-1)}{n} = \frac{4}{n}$$

5. The interval  $[-1, 3]$  is partitioned into  $n$  subintervals of equal length  $\Delta x = \frac{4}{n}$ . Let  $m_k$  denote the midpoint of the  $k^{\text{th}}$  subinterval. Express the limit

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n (3(m_k)^2 - 2m_k + 5)\Delta x \quad f(m_k) = 3(m_k)^2 - 2m_k + 5$$

as an integral.

$$f(x) = 3x^2 - 2x + 5$$

$$\int_{-1}^3 (3x^2 - 2x + 5) dx$$

5. The function  $f$  is given by  $f(x) = \ln x$ . The graph of  $f$  is shown below. Which of the following is equal to the area of the shaded region?

(A)  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \ln\left(\frac{3k}{n}\right)\right) \frac{3}{n}$  width of rectangles  $= \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$

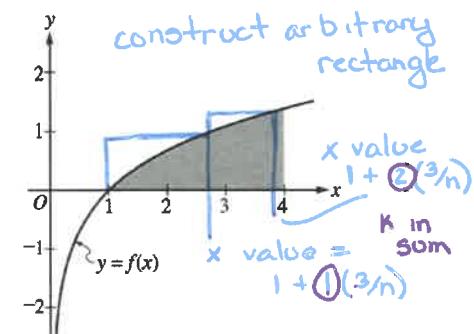
(B)  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \ln\left(1 + \left(\frac{3k}{n}\right)\right) \frac{3}{n}$  height  $= f(a + k\Delta x) = f(1 + k(3/n)) = \ln(1 + \frac{3k}{n})$

(C)  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \ln\left(\frac{4}{n}\right) \left(1 + \frac{4k}{n}\right)$   $\cancel{\text{K starts @ 1 in sum}}$   
 $\cancel{\text{want right to start @ a+width so a+k\Delta x}}$   
 $\cancel{\text{want left to start @ a so a+(k-1)\Delta x}}$   
 $\cancel{\text{* k=0 be starting point}}$

6. The closed interval  $[a, b]$  is partitioned into  $n$  subintervals each of width  $\Delta x$ , by the numbers

~~equally~~  $x_0, x_1, \dots, x_n$  where  $a = x_0 < x_1 < \dots < x_{n-1} < x_n = b$ . Express  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{x_i} \Delta x$  as an integral.

$$\int_a^b \sqrt{x} dx$$



\* right Riemann sum  
 $f(a + k\Delta x)$   
 left Riemann sum  
 $f(a + (k-1)\Delta x)$

7. For a certain continuous function,  $f$ , the right Riemann sum approximation of  $\int_0^2 f(x)dx$  with  $n$  subintervals

of equal length is  $\frac{2(n+1)(3n+2)}{n^2}$  for all  $n$ . What is the value of  $\int_0^2 f(x)dx$ ?

- A 2     C 12  
 B 6     D 20

Riemann sum  $\rightarrow S \star \lim_{n \rightarrow \infty}$

$$\lim_{n \rightarrow \infty} \cdot \frac{2(n+1)(3n+2)}{n^2} = \frac{6}{1} = 6$$

8. Which of the following limits is equal to  $\int_2^5 x^2 dx$

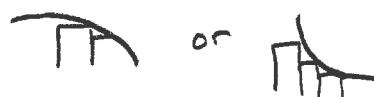
$$\text{width} = \frac{5-2}{n} = \frac{3}{n}$$

- A  $\lim_{x \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{k}{n}\right)^2 \frac{1}{n}$      C  $\lim_{x \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{3k}{n}\right)^2 \frac{1}{n}$     no  $n-1$  so right Riemann  
 B  $\lim_{x \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{k}{n}\right)^2 \frac{3}{n}$      D  $\lim_{x \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{3k}{n}\right)^2 \frac{3}{n}$   
height =  $f(a + k \Delta x)$   
=  $f(2 + k(3/n))$   
=  $(2 + \frac{3k}{n})^2$

9. The continuous function  $f$  is decreasing for all  $x$ . Selected values of  $f$  are given in the table below, where  $a$  is a constant with  $0 < a < 3$ . Let  $R$  be the ~~right~~ Riemann sum approximation for  $\int_0^{7a} f(x)dx$  using four subintervals indicated by the data in the table. Which of the following statements is true?

$x$	0	$a^2$	$3a$	$6a$	$7a$
$f(x)$	1	-1	-3	-7	-9

decreasing:



both underestimates

- A  $R = (a^2 - 0) \cdot 1 + (3a - a^2) \cdot (-1) + (6a - 3a) \cdot (-3) + (7a - 6a) \cdot (-7)$  and is an underestimate for  $\int_0^{7a} f(x) dx$ .  
left endpoint of rectangle 1    LRAM

- B  $R = (a^2 - 0) \cdot 1 + (3a - a^2) \cdot (-1) + (6a - 3a) \cdot (-3) + (7a - 6a) \cdot (-7)$  and is an overestimate for  $\int_0^{7a} f(x) dx$ .

- C  $R = (a^2 - 0) \cdot (-1) + (3a - a^2) \cdot (-3) + (6a - 3a) \cdot (-7) + (7a - 6a) \cdot (-9)$  and is an underestimate for  $\int_0^{7a} f(x) dx$ .  
right endpoint of rectangle 1    RRAM

- D  $R = (a^2 - 0) \cdot (-1) + (3a - a^2) \cdot (-3) + (6a - 3a) \cdot (-7) + (7a - 6a) \cdot (-9)$  and is an overestimate for  $\int_0^{7a} f(x) dx$ .

10) which of the following is a left Riemann sum approximation  
of  $\int_2^8 \cos(x^2) dx$  w/ n subintervals of equal length?

$$\text{width of rectangle} = \frac{8-2}{n} = \frac{6}{n}$$



$$\text{height} = f(a + (k-1)\Delta x)$$

↑  
K-1 b/c want to start @ a and sum  
has  $k=1$

\*left Riemann\*

$$= f(2 + (k-1)\frac{6}{n})$$

$$= \cos\left(2 + \frac{6(k-1)}{n}\right)$$

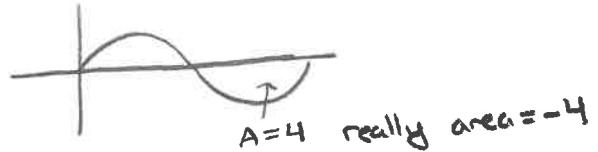
c)  $\sum_{k=1}^n \left[ \cos\left(2 + \frac{6(k-1)}{n}\right)^2 \right] \frac{6}{n}$

**Area Under a Curve**

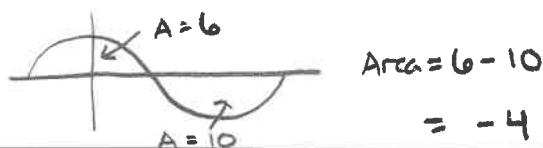
If  $y = f(x)$  is nonnegative and integrable over a closed interval  $[a, b]$ , then the area under the curve  $y = f(x)$  from  $a$  to  $b$  is the integral of  $f$  from  $a$  to  $b$ ,

$$A = \int_a^b f(x) dx$$

When  $f(x) \leq 0$  Area =  $-\int_a^b f(x) dx$

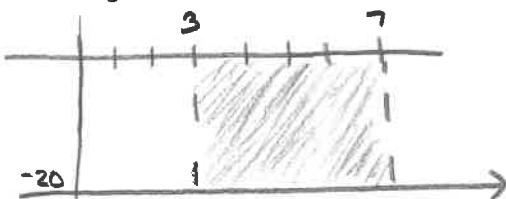


$$\int_a^b f(x) dx = (\text{area above } x\text{-axis}) - (\text{area below } x\text{-axis})$$



10. Evaluate the integral below:

a.  $\int_3^7 (-20) dx$



$$\text{Area} = \int_3^7 (-20) dx = 4(-20)$$

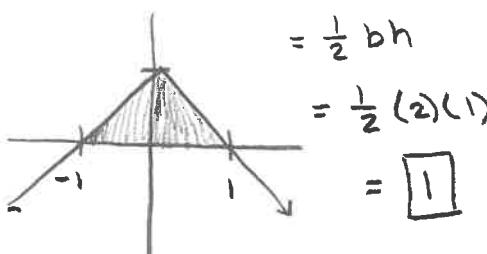
$$= \boxed{-80}$$

11. Use the graph of the integrand and area to evaluate the integral:

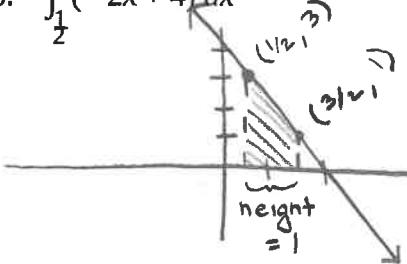
a.  $\int_{-1}^1 (1 - |x|) dx$

$$= \frac{1}{2}bh$$

$$= \frac{1}{2}(2)(1)$$



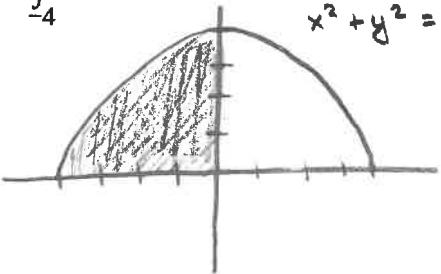
b.  $\int_{\frac{1}{2}}^3 (-2x + 4) dx$



$$\int_{\frac{1}{2}}^3 (-2x + 4) dx = \frac{1}{2}(1)(3 + 1)$$

$$= \boxed{2}$$

c.  $\int_{-4}^0 \sqrt{16 - x^2} dx$



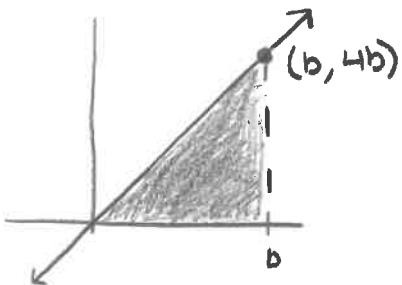
$$\int_{-4}^0 \sqrt{16 - x^2} dx = \frac{1}{4}\pi r^2$$

$$= \frac{1}{4}\pi (4)^2$$

$$= \boxed{4\pi}$$

12. Use areas to evaluate the integral:

a.  $\int_0^b 4x \, dx, \quad b > 0$

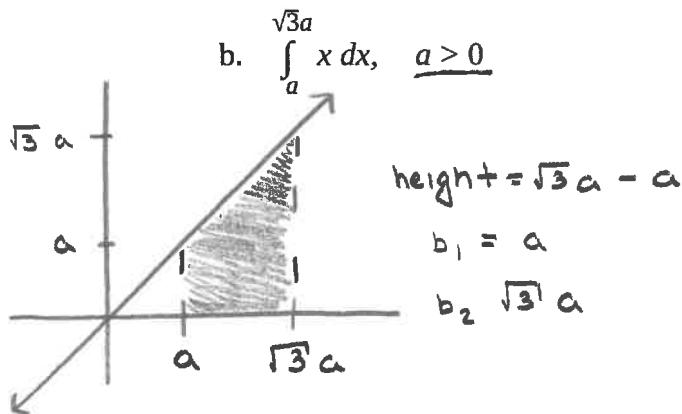


$$= \frac{1}{2}(\text{base})(\text{height})$$

$$= \frac{1}{2} b (4b)$$

$$= \boxed{2b^2}$$

b.  $\int_a^{\sqrt{3}a} x \, dx, \quad a > 0$



$$= \frac{1}{2} (\sqrt{3}a - a)(a + \sqrt{3}a)$$

$$= \frac{1}{2} (\sqrt{3}a^2 - a^2 + 3a^2 - \sqrt{3}a^2)$$

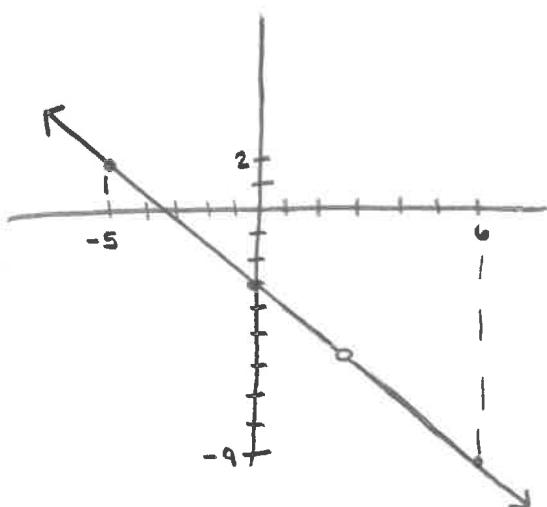
$$= \frac{1}{2} (2a^2)$$

$$= \boxed{a^2}$$

13. Find the points of discontinuity of the integrand on the interval of integration, and use area to evaluate the integral.

a.  $\int_{-5}^6 \frac{9-x^2}{x-3} \, dx$

$$\frac{-(x^2-9)}{x-3} = \frac{-(x-3)(x+3)}{x-3} = -(x+3) = -x-3, \quad x \neq 3$$



$$\int_{-5}^6 \frac{9-x^2}{x-3} \, dx = \frac{1}{2} (2)(2) + \frac{1}{2} (9)(-9)$$

$$= 2 - \frac{81}{2}$$

$$= \boxed{-\frac{77}{2}}$$