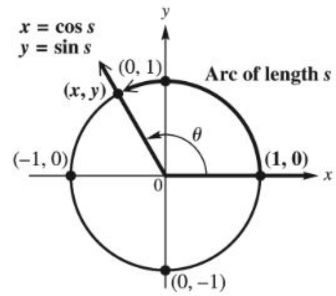
Circular Functions

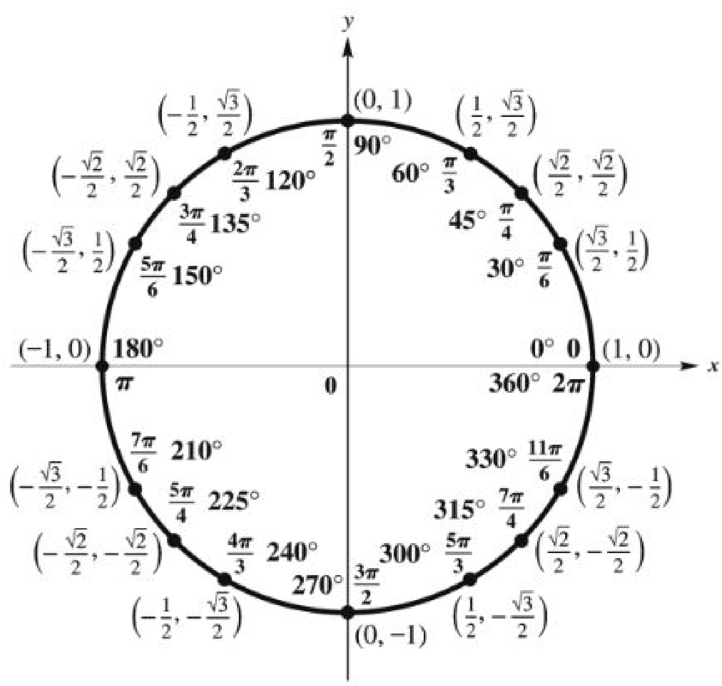


The unit circle $x^2 + y^2 = 1$

Circular Functions
The following functions are defined for any real number s represented by a directed arc on the unit circle.

$$\sin s = y \qquad \cos s = x \qquad \tan s = \frac{y}{x}\ (x \neq 0)$$

$$\csc s = \frac{1}{y}\ (y \neq 0) \qquad \sec s = \frac{1}{x}\ (x \neq 0) \qquad \cot s = \frac{x}{y}\ (y \neq 0)$$



The unit circle $x^2 + y^2 = 1$

The unit circle is symmetric with respect to the ___ ________, the ___ ________, *and the* ____________. If a point (a, b) lies on the unit circle, so do ____________, ____________ and ____________. Furthermore, each of these points has a ____________ ____________of equal magnitude.

Because $\cos s = x$ and $\sin s = y$, we can replace x and y in the equation of the unit circle, $x^2 + y^2 = 1$ and obtain the following.

$$\cos^2 s + \sin^2 s = 1$$

Domains of the Circular Functions
The domains of the circular functions are as follows.

Sine and Cosine Functions: $(-\infty, \infty)$

Tangent and Secant Functions:

$$\left\{ s \mid s \neq (2n+1)\frac{\pi}{2}, \text{ where } n \text{ is any integer} \right\}$$

Cotangent and Cosecant Functions:

$$\{ s \mid s \neq n\pi, \text{ where } n \text{ is any integer} \}$$

1. Find the following:
   a. $\sin(-3\pi)$
   b. $\cot(\frac{3\pi}{2})$
   c. $\sin(\frac{11\pi}{6})$
   d. $\cos(\frac{4\pi}{3})$
   e. $\sin(\frac{7\pi}{6})$
   f. $\tan(\frac{-9\pi}{4})$

2. Find a calculator approximation for each of the following:

(a) $\sin 3.42$

(b) $\tan 0.8234$

(c) $\sec 5.6041$

(d) $\csc(-2.7335)$

Remember, when used to find a circular function value of a real number, the calculator must be in _____________ mode.

Determining a Number with a Given Circular Function Value

The keys marked $\sin^{-1}$, $\cos^{-1}$, **and** $\tan^{-1}$ ***do not represent reciprocal functions. They enable us to find inverse function values.***

3. Find each value as specified:

(a) Approximate the value of s in the interval $\left[0, \frac{\pi}{2}\right]$ if $\cos s = 0.3210$.

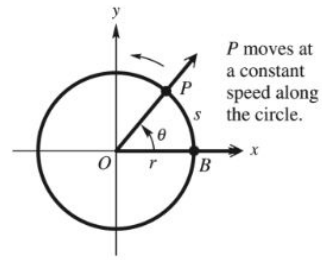
(b) Find the exact value of s in the interval $\left[\frac{3\pi}{2}, 2\pi\right]$ if $\tan s = -\frac{\sqrt{3}}{3}$.

Linear and Angular Speed

Suppose that point P moves at a constant speed along a circle of radius r and center O. The measure of how fast the position of P is changing is the **linear speed**. If v represents linear speed, then

$$\text{speed} = \frac{\text{distance}}{\text{time}}, \quad \text{or} \quad \boldsymbol{v} = \frac{\boldsymbol{s}}{\boldsymbol{t}},$$

where s is the length of the arc traced by point P at time t.



As point P in the figure moves along the circle, ray OP rotates around the origin. Because ray OP is the terminal side of angle POB, the measure of the angle changes as P moves along the circle. The measure of how fast angle POB is changing is its **angular speed**. Angular speed, symbolized ω, is given as

$$\boldsymbol{\omega = \frac{\theta}{t},} \quad \textbf{where } \boldsymbol{\theta} \textbf{ is in radians.}$$

Here θ is the measure of angle POB at time t. ***As with earlier formulas in this chapter, θ must be measured in __________, with ω expressed in __________ __________ __________ __________ __________.***

Formulas for Angular and Linear Speed

Angular Speed ω	Linear Speed v
$\omega =$ ______	$v =$ ______
(ω in radians per unit time t, θ in radians)	$v =$ ______
	$v =$ ______

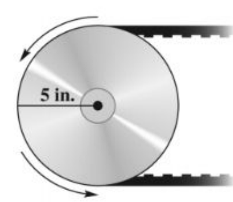
4. Suppose that point P is on a circle with radius 15 in., and ray OP is rotating with angular speed $\frac{\pi}{12}$ radian per sec.
   (a) Find the angle generated by P in 10 sec.

(b) Find the distance traveled by *P* along the circle in 10 sec.

(c) Find the linear speed of *P* in inches per second.

5. A belt runs a pulley of radius 5 in. at 120 revolutions per min.

(a) Find the angular speed of the pulley in radians per second.



(b) Find the linear speed of the belt in inches per second.