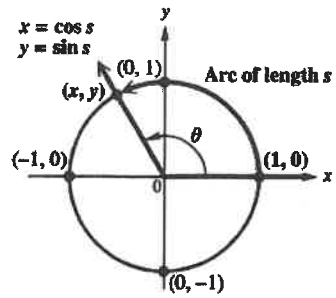
Circular Functions

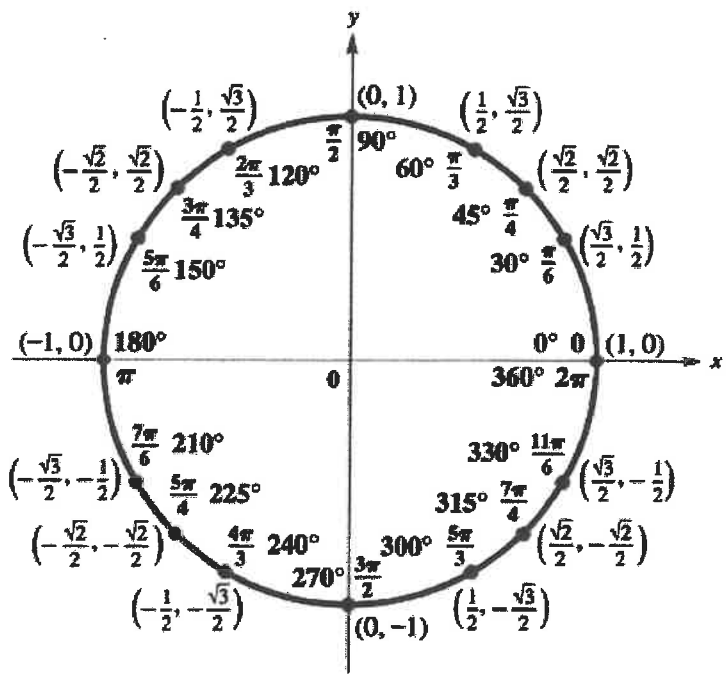


The unit circle $x^2 + y^2 = 1$

Circular Functions
The following functions are defined for any real number s represented by a directed arc on the unit circle.

$\sin s = y$ $\quad$ $\cos s = x$ $\quad$ $\tan s = \frac{y}{x}\ (x \neq 0)$

$\csc s = \frac{1}{y}\ (y \neq 0)$ $\quad$ $\sec s = \frac{1}{x}\ (x \neq 0)$ $\quad$ $\cot s = \frac{x}{y}\ (y \neq 0)$



The unit circle $x^2 + y^2 = 1$

centered at (0,0) w/ radius = 1

The unit circle is symmetric with respect to the x axis**, the** y axis**, *and the*** origin**. If a point (a, b) lies on the unit circle, so do** (a, -b)**,** (-a, b) **and** (-a, -b)**. Furthermore, each of these points has a** reference arc **of equal magnitude.**

Because $\cos s = x$ and $\sin s = y$, we can replace x and y in the equation of the unit circle, $x^2 + y^2 = 1$ and obtain the following.

$$\cos^2 s + \sin^2 s = 1$$

Domains of the Circular Functions
The domains of the circular functions are as follows.

Sine and Cosine Functions: $(-\infty, \infty)$

Tangent and Secant Functions:

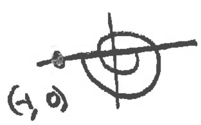
$$\left\{ s \mid s \neq (2n+1)\frac{\pi}{2}, \text{ where } n \text{ is any integer} \right\}$$

Cotangent and Cosecant Functions:

$$\{ s \mid s \neq n\pi, \text{ where } n \text{ is any integer} \}$$

1. Find the following:

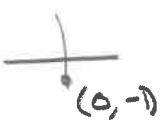
a. $\sin(-3\pi)$ $= 0$



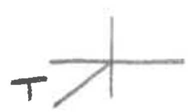
d. $\cos(\frac{4\pi}{3})$ $= -\frac{1}{2}$

$\theta' = \pi/3 = 60°$

b. $\cot(\frac{3\pi}{2})$ $= 0$



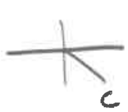
e. $\sin(\frac{7\pi}{6})$ $= -\frac{1}{2}$



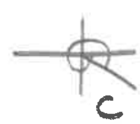
$\theta' = \pi/6 = 30°$

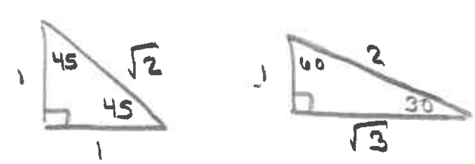
c. $\sin(\frac{11\pi}{6})$ $= -\frac{1}{2}$

$\theta' = \frac{\pi}{6} = 30°$



f. $\tan(\frac{-9\pi}{4})$ $= -1$





2. Find a calculator approximation for each of the following:

(a) $\sin 3.42$

$= -0.275$

(b) $\tan 0.8234$

$= 1.0790$

(c) $\sec 5.6041$

$= 1.285$

(d) $\csc(-2.7335)$

$= -2.5198$

Remember, when used to find a circular function value of a real number, the calculator must be in radian ***mode.***

Determining a Number with a Given Circular Function Value

The keys marked **$\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$** ***do not represent reciprocal functions. They enable us to find inverse function values.***

☆ $\sin^{-1} x \neq \frac{1}{\sin x}$

means inverse

3. Find each value as specified:

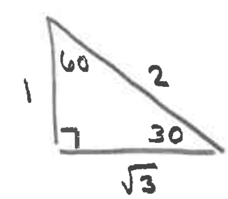
(a) Approximate the value of s in the interval $\left[0, \frac{\pi}{2}\right]$ if $\cos s = 0.3210$.

$s = \cos^{-1}(0.3210)$

$s = 1.244$

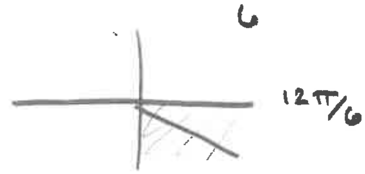
(b) Find the exact value of s in the interval $\left[\frac{3\pi}{2}, 2\pi\right]$ if $\tan s = -\frac{\sqrt{3}}{3}$.

$s = \tan^{-1}(-\sqrt{3}/3)$



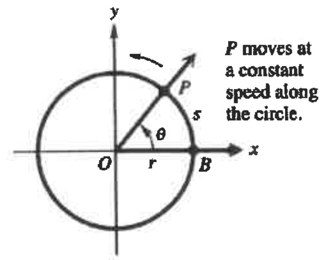
$s = 11\pi/6$

$\theta' = 30^\circ$
$= \frac{\pi}{6}$



Linear and Angular Speed

Suppose that point P moves at a constant speed along a circle of radius r and center O. The measure of how fast the position of P is changing is the **linear speed.** If v represents linear speed, then



$$\text{speed} = \frac{\text{distance}}{\text{time}}, \quad \text{or} \quad v = \frac{s}{t},$$

where s is the length of the arc traced by point P at time t.

As point P in the figure moves along the circle, ray OP rotates around the origin. Because ray OP is the terminal side of angle POB, the measure of the angle changes as P moves along the circle. The measure of how fast angle POB is changing is its **angular speed**. Angular speed, symbolized ω, is given as

$$\omega = \frac{\theta}{t}, \quad \textbf{where } \theta \textbf{ is in radians.}$$

Here θ is the measure of angle POB at time t. ***As with earlier formulas in this chapter,*** θ ***must be measured in*** radians, ***with*** ω ***expressed in*** radians per unit of time.

Formulas for Angular and Linear Speed

Angular Speed ω	Linear Speed v
$\omega = \frac{\theta}{t}$	$v = \frac{s}{t}$
(ω in radians per unit time t, θ in radians)	$v = \frac{r\theta}{t}$
	$v = r\omega$

4. Suppose that point P is on a circle with radius 15 in., and ray OP is rotating with angular speed $\frac{\pi}{12}$ radian per sec.

$r = 15 \text{ in}$

$w = \pi/12$

(a) Find the angle generated by P in 10 sec.

$$w = \frac{\theta}{t}$$

$$\frac{\pi}{12} = \frac{\theta}{10}$$

$$\boxed{\frac{5\pi}{6} = \theta}$$

s

(b) Find the distance traveled by P along the circle in 10 sec.

$v = \frac{s}{t}$ $v = r\omega$

$r\omega = \frac{s}{t}$

$15 \cdot \frac{\pi}{12} = \frac{s}{10}$ $\boxed{\frac{25}{2}\pi \text{ in} = s}$

(c) Find the linear speed of P in inches per second.

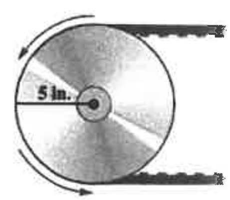
$v = r\omega$

$v = 15 \cdot \frac{\pi}{12}$

$\boxed{v = \frac{5\pi}{4} \text{ in. per sec}}$

5. A belt runs a pulley of radius 5 in. at 120 revolutions per min.

(a) Find the angular speed of the pulley in radians per second.



1 revolution = 2π radians 1 min = 60 sec

$\omega = \frac{120 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 4\pi$

$\boxed{4\pi \text{ radians}}$

(b) Find the linear speed of the belt in inches per second.

$v = r\omega$

$v = 5(4\pi)$

$= \boxed{20\pi \text{ in per sec}}$

≈ 63 in per sec