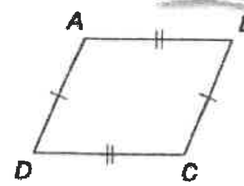
## Theorems — Conditions for Parallelograms

For Your FOLDABLE

**6.9** If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

**Abbreviation** *If both pairs of opp. sides are ≅, then quad. is a ▱.*

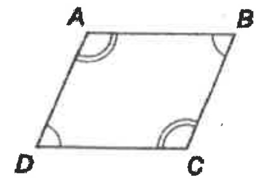
**Example** If $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$, then $ABCD$ is a parallelogram.




**6.10** If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

**Abbreviation** *If both pairs of opp. ∠s are ≅, then quad. is a ▱.*

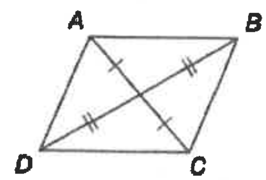
**Example** If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.




**6.11** If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

**Abbreviation** *If diag. bisect each other, then quad. is a ▱.*

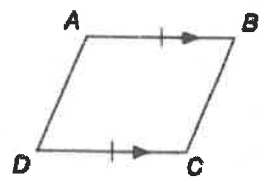
**Example** If $\overline{AC}$ and $\overline{DB}$ bisect each other, then $ABCD$ is a parallelogram.




**6.12** If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

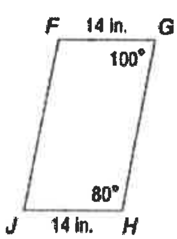
**Abbreviation** *If one pair of opp. sides is ≅ and ||, then the quad. is a ▱.*

**Example** If $\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$, then $ABCD$ is a parallelogram.



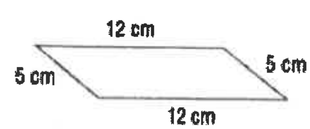

1. Determine whether the quadrilateral is a parallelogram. Justify your answer.

a.




yes b/c consecutive interior angles are supplemetary which means $\overline{FG} \parallel \overline{JH}$ by consecutive int. ∠ converse thm. There is one pair of opposite ≅ sides and they are parallel so by Thm 6.12 FGHJ is a ▱.

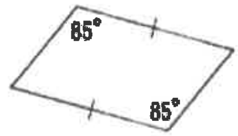
b.




yes b/c both pairs of opposite sides are congruent

Thm 6.9

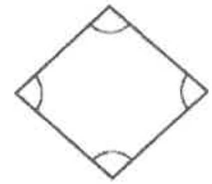
c.




no

b/c both pairs of opposite <'s are not ≅ and both pairs of opposite <'s are not ≅

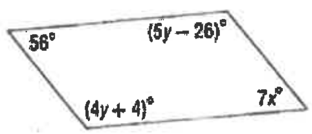
d.



yes b/c both pairs of opposite angles are ≅ (Thm 6.10)

2. Find the variables in the diagrams below so that each quadrilateral is a parallelogram.

a.



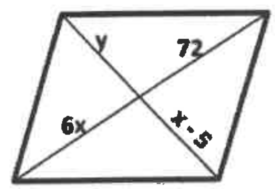

$56 = 7x$

$\boxed{8° = x}$

$5y - 26 = 4y + 4$

$\boxed{y = 30°}$

c.




$72 = 6x$

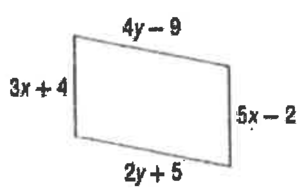
$\boxed{12° = x}$

$y = x - 5$

$y = 12 - 5$

$\boxed{y = 7°}$

b.




$3x + 4 = 5x - 2$

$6 = 2x$

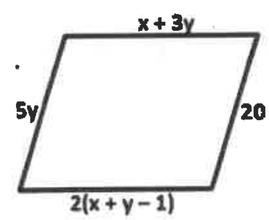
$\boxed{3° = x}$

$4y - 9 = 2y + 5$

$2y = 14$

$\boxed{y = 7°}$

d.




$5y = 20$

$\boxed{y = 4°}$

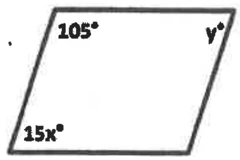
$x + 3y = 2(x + y - 1)$

$x + 3(4) = 2x + 2(4) - 2$

$x + 12 = 2x + 6$

$\boxed{6° = x}$

e.




$105 + 15x = 180$

$15x = 75$

$\boxed{x = 5°}$

$105 + y = 180$

$\boxed{y = 75°}$

3. Show that $A(2,-1)$, $B(1,3)$, $C(6,5)$, and $D(7,1)$ are the vertices of a parallelogram.


Method 1: Show that the opposite sides have the same

length

$d_{AB} = \sqrt{(2-1)^2 + (-1-3)^2} = \sqrt{1+16} = \sqrt{17}$

$d_{CD} = \sqrt{(6-7)^2 + (5-1)^2} = \sqrt{1+16} = \sqrt{17}$

$d_{BC} = \sqrt{(1-6)^2 + (3-5)^2} = \sqrt{25+4} = \sqrt{29}$

$d_{AD} = \sqrt{(2-7)^2 + (-1-1)^2} = \sqrt{25+4} = \sqrt{29}$

$\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$

Method 2: Show that the opposite sides have the same slope

$m_{AB} = \dfrac{3-(-1)}{1-2} = -4$

$m_{CD} = \dfrac{5-1}{6-7} = -4$

$\overline{AB} \parallel \overline{CD}$

$m_{BC} = \dfrac{3-5}{1-6} = \dfrac{-2}{-5} = \dfrac{2}{5}$

$m_{AD} = \dfrac{-1-1}{2-7} = \dfrac{2}{5}$

$\overline{BC} \parallel \overline{AD}$

Method 3: Show that one pair of opposite sides is congruent and parallel

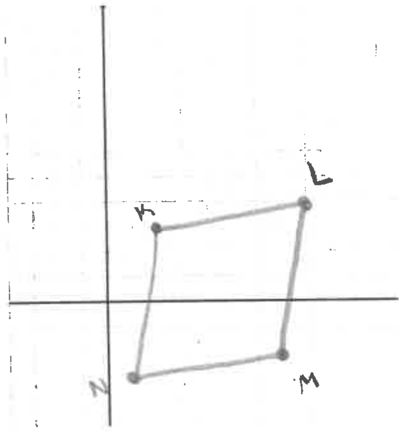
$AB = \sqrt{17} = CD$

$\overline{AB} \cong \overline{CD}$

$m_{AB} = -4 = m_{CD}$

$\overline{AB} \parallel \overline{CD}$

4. Graph quadrilateral KLMN with vertices $K(2,3)$, $L(8,4)$, $M(7,-2)$, and $N(1,-3)$. Determine whether the quadrilateral is a parallelogram.




$d_{KL} = \sqrt{(2-8)^2 + (3-4)^2} = \sqrt{36+1} = \sqrt{37}$

$d_{NM} = \sqrt{(1-7)^2 + (-3-(-2))^2} = \sqrt{36+1} = \sqrt{37}$

$m_{KL} = \frac{3-4}{2-8} = \frac{1}{6}$ $m_{NM} = \frac{-3-(-2)}{1-7} = \frac{1}{6}$

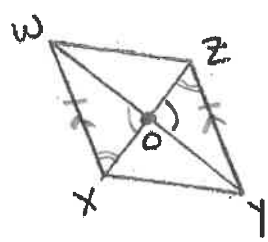
$\overline{KL} \cong \overline{NM}$ & $\overline{KL} \parallel \overline{NM}$

Thm 6.12 KLMN is a ▱

5.

Given: ▱ WXYZ

Prove: $\Delta WOX \cong \Delta YOZ$

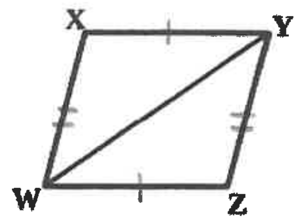



| Statements | Justifications |
|---|---|
| 1. ▱ WXYZ | 1. Given. |
| 2. $\angle WOX \cong \angle ZOY$ | 2. Vertical Angles |
| 3. $\overline{XW} \parallel \overline{YZ}$ | 3. Definition of Parallelogram |
| 4. $\angle WXO \cong \angle YZO$ | 4. Alt Int <'s Thm |
| 5. $\overline{WX} \cong \overline{YZ}$ | 5. Thm 6.3 |
| 6. $\Delta WOX \cong \Delta YOZ$ | 6. AAS |

4.

**Given:** $\Delta XYW \cong \Delta ZWY$

**Prove:** XYZW is a parallelogram.

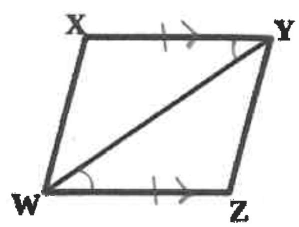



| Statements | Justifications |
|---|---|
| 1. $\Delta XYW \cong \Delta ZWY$ | 1. Given |
| 2. $\overline{XY} \cong \overline{WZ}$ | 2. CPCTC |
| 3. $\overline{XW} \cong \overline{YZ}$ | 3. CPCTC |
| 4. XYZW is a parallelogram | 4. opposite sides are $\cong$ Thm 6.9 |

5.

**Given:** $\Delta XYW \cong \Delta ZWY$

**Prove:** XYZW is a parallelogram.




| Statements | Justifications |
|---|---|
| 1. $\Delta XYW \cong \Delta ZWY$ | 1. Given |
| 2. $\measuredangle XYW \cong \measuredangle YWZ$ | 2. CPCTC |
| 3. $\overline{XY} \parallel \overline{WZ}$ | 3. Alt Int < Converse Thm |
| 4. $\overline{XY} \cong \overline{WZ}$ | 4. CPCTC |
| 5. XYZW is a parallelogram | 5. one pair of opposite sides are $\cong$ & parallel (Thm 6.12) |