Table 5.3	**Rules for Definite Integrals**	
1. *Order of Integration:*	$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$	A definition
2. *Zero:*	$\int_a^a f(x)\,dx = 0$	Also a definition
3. *Constant Multiple:*	$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$	Any number k
	$\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$	$k = -1$
4. *Sum and Difference:*	$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$	
5. *Additivity:*	$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$	

1. Suppose that f and g are continuous functions and that

$$\int_1^2 f(x)dx = -4, \quad \int_1^5 f(x)dx = 6, \quad \int_1^5 g(x)dx = 8$$

Find each integral below:

a. $\int_2^2 g(x)dx$

d. $\int_2^5 f(x)dx$

b. $\int_5^1 g(x)dx$

e. $\int_1^5 [f(x) - g(x)]dx$

c. $\int_1^2 3f(x)dx$

f. $\int_1^5 [4f(x) - g(x)]dx$

2. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x)dx$?

(A) 2

(B) 6

(C) 8

(D) 10

(E) 12

3. Let f and g have continuous first and second derivatives everywhere. If $f(x) \le g(x)$ for all real x, which of the following must be true?

   I. $f'(x) \le g'(x)$ for all real x.

   II. $f''(x) \le g''(x)$ for all real x.

   III. $\int_0^1 f(x)dx \le \int_0^1 g(x)dx$

   A none

   B I only

   C III only

   D I and II only

   E I, II, and III

4. The function f is defined as $f(x) = \begin{cases} \frac{|x|}{x} & \text{for } x \ne 0 \\ 0 & \text{for } x = 0 \end{cases}$ The value $\int_{-5}^{3} 5f(x)dx$ is:

   A -2

   B 2

   C 8

   D nonexistent

5. Interpret the integrand as the rate of change of a quantity and evaluate the integral using the antiderivative of the quantity:

   a. $\int_0^{\frac{\pi}{2}} \cos x \, dx$

   b. $\int_0^{\frac{\pi}{4}} \sec^2 x \, dx$

   c. $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx$

d. $\int_{-1}^{2} 3x^2 \, dx$

e. $\int_{3}^{7} 8 \, dx$

f. $\int_{1}^{4} -x^{-2} \, dx$

5. If f is a linear function and $0 < a < b$, then $\int_{a}^{b} f''(x)dx =$

(A) 0

(B) 1

(C) $\frac{ab}{2}$

(D) b-a

(E) $\frac{b^2-a^2}{2}$

DEFINITION Average (Mean) Value

If f is integrable on $[a, b]$, its **average (mean) value** on $[a, b]$ is

$$av(f) = \frac{1}{b-a}\int_{a}^{b} f(x)\, dx.$$

6. Find the average value of the function on the interval, using antiderivatives to compute the integral.

a. $y = \frac{1}{x}, \quad [e, 2e]$

b. $y = \frac{1}{1+x^2}, \quad [0, 1]$

c. $y = \sec x \tan x, \quad [0, \frac{\pi}{3}]$