

6.3 Definite Integrals and Antiderivative

Table 5.3 Rules for Definite Integrals

1. Order of Integration: $\int_b^a f(x) dx = - \int_a^b f(x) dx$ A definition

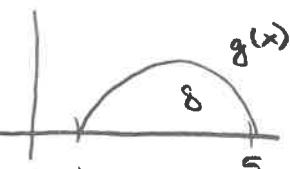
2. Zero: $\int_a^a f(x) dx = 0$ Also a definition

3. Constant Multiple: $\int_a^b kf(x) dx = k \int_a^b f(x) dx$ Any number k

$$\int_a^b -f(x) dx = - \int_a^b f(x) dx \quad k = -1$$

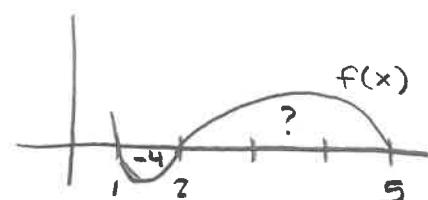
4. Sum and Difference: $\int_a^b (f(x) \pm g(x)) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$

5. Additivity: $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$

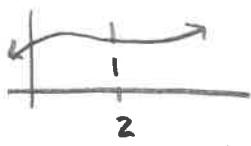


1. Suppose that f and g are continuous functions and that

$$\int_1^2 f(x) dx = -4, \quad \int_1^5 f(x) dx = 6, \quad \int_1^5 g(x) dx = 8$$



Find each integral below:



a. $\int_2^2 g(x) dx = 0$

No area to find

b. $\int_5^1 g(x) dx = -8$

c. $\int_1^2 3f(x) dx = 3 \int_1^2 f(x) dx$

$$= -12$$

2. The function f is defined by

$$\int_1^5 f(x) dx?$$

A

2

C 8

B 6

D 10

E 12

$2 + \frac{16}{2}$

$2 + 8$

$$f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$$

What is the value of

$$\int_1^3 f(x) dx + \int_3^5 f(x) dx$$

$$\int_1^3 2 dx + \int_3^5 (x-1) dx$$

$$2x \Big|_1^3 + \left(\frac{1}{2}x^2 - x \right) \Big|_3^5$$

$$2(3) - 2 + \left[\left(\frac{1}{2}(5)^2 - 5 \right) - \left(\frac{1}{2}(3)^2 - 3 \right) \right]$$

$$6 - 2 + \frac{25}{2} - 5 - \frac{9}{2} + 3$$

* simple examples

of f and g

↳ g could have part below x -axis

d. $\int_2^5 f(x) dx = 10$

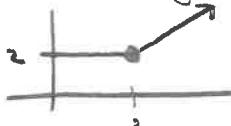
e. $\int_1^5 [f(x) - g(x)] dx$

$$= 6 - 8 = -2$$

f. $\int_1^5 [4f(x) - g(x)] dx = 4(6) - 8$

$$= 16$$

OR compute geometrically



6.3 Definite Integrals and Antiderivative

3. Let f and g have continuous first and second derivatives everywhere. If $f(x) \leq g(x)$ for all real x , which of the following must be true?

- I. $f'(x) \leq g'(x)$ for all real x . → don't know how slopes compare
 II. $f''(x) \leq g''(x)$ for all real x . → maybe:
 III. $\int_0^1 f(x)dx \leq \int_0^1 g(x)dx$ don't know how concavity compares
 maybe:
 III only
-
- A** none **D** I and II only
B I only **E** I, II, and III

4. The function f is defined as

$$f(x) = \begin{cases} \frac{|x|}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

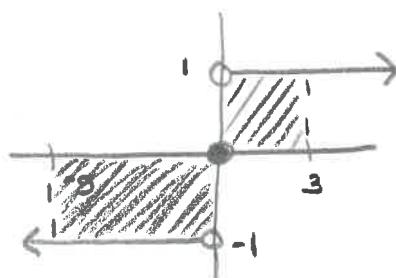
The value $\int_{-5}^3 5f(x)dx$ is:

A -2

B 16

C -10

D nonexistent



$$= 5(5(-1)) + 5(3(1))$$

$$= -25 + 15$$

$$= -10$$

5. Interpret the integrand as the rate of change of a quantity and evaluate the integral using the antiderivative of the quantity:

a. $\int_0^{\pi} \cos x dx$

$$= \sin x \Big|_0^{\pi/2}$$

$$= \sin \pi/2 - \sin 0$$

$$= 1 - 0$$

$$= 1$$

b. $\int_0^{\pi} \sec^2 x dx$

$$= \tan x \Big|_0^{\pi/4}$$

$$= \tan \pi/4 - \tan 0$$

$$= 1 - 0$$

$$= 1$$

c. $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx$

$$= \sin^{-1} x \Big|_0^{\pi/2}$$

$$= \sin^{-1} \pi/2 - \sin^{-1} 0$$

$$= \frac{\pi}{6} - 0$$

$$= \frac{\pi}{6}$$

6.3 Definite Integrals and Antiderivative

d. $\int_{-1}^2 3x^2 dx$

$$= \frac{3}{3} x^3 \Big|_{-1}^2$$

$$= 2^3 - (-1)^3$$

$$= 8 - (-1)$$

$$= 9$$

e. $\int_3^7 8 dx$

$$= 8x \Big|_3^7$$

$$= 8(7) - 8(3)$$

$$= 56 - 24$$

$$= 32$$

f. $\int_1^4 -x^{-2} dx$

$$= -\frac{x^{-1}}{-1} \Big|_1^4$$

$$= 4^{-1} - 1^{-1}$$

$$= \frac{1}{4} - 1$$

$$= -\frac{3}{4}$$

5. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)dx =$

can think of it as:

$\int_a^b f''(x)dx = f'(x) \Big|_a^b$

(C) $\frac{ab}{2}$

 $f \rightarrow \text{linear}$

(D) $b-a$

 $f' \rightarrow \text{constant}$

$\int_a^b 0 dx$
 $= 0$

 $f' \text{ constant}$

(B) 0

(E) $\frac{b^2-a^2}{2}$

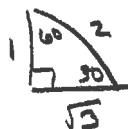
 $f'' \rightarrow 0$

$f'(a) = f'(b)$

$f'(b) - f'(a) = 0$

DEFINITION Average (Mean) ValueIf f is integrable on $[a, b]$, its **average (mean) value** on $[a, b]$ is

$$av(f) = \frac{1}{b-a} \int_a^b f(x) dx.$$



6. Find the average value of the function on the interval, using antiderivatives to compute the integral.

a. $y = \frac{1}{x}$, $[e, 2e]$

$$\frac{1}{2e-e} \int_e^{2e} \frac{1}{x} dx$$

$$\frac{1}{e} \ln|x| \Big|_e^{2e}$$

$$\frac{1}{e} (\ln 2e - \ln e)$$

$$\frac{1}{e} \ln \frac{2e}{e}$$

$$\frac{1}{e} \ln 2$$

b. $y = \frac{1}{1+x^2}$, $[0, 1]$

$$\frac{1}{1-0} \int_0^1 \frac{1}{1+x^2} dx$$

$$\tan^{-1} x \Big|_0^1$$

$$\tan^{-1}(1) - \tan^{-1}(0)$$

$$\frac{\pi}{4} - 0$$

$$\frac{\pi}{4}$$

c. $y = \sec x \tan x$,

$$[0, \frac{\pi}{3}] \int_0^{\pi/3} \sec x \tan x dx$$

$$\frac{3}{\pi} \sec x \Big|_0^{\pi/3}$$

$$\frac{3}{\pi} (\sec \pi/3 - \sec 0)$$

$$\frac{3}{\pi} (2 - 1)$$

$$\frac{3}{\pi}$$

