THEOREM 4 The Fundamental Theorem of Calculus, Part 1

If f is continuous on $[a, b]$, then the function

$$F(x) = \int_a^x f(t)\, dt$$

has a derivative at every point x in $[a, b]$, and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\, dt = f(x).$$

THEOREM 4 (continued) The Fundamental Theorem of Calculus, Part 2

If f is continuous at every point of $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

This part of the Fundamental Theorem is also called the **Integral Evaluation Theorem.**

$$\frac{d}{dx}\int_a^x f(t)\, dt = f(x). \qquad (1)$$

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

1. If $g(x) = \int_{-2}^{x} w(t)dt$, then $g'(x) = ?$

2. $\frac{d}{dx}\left[\int_{3}^{x}(5t^2 - 6t + 1)dt\right]$

3. Let $g(x) = \int_{0}^{3x^2} \sin(t)\, dt$. What is $g'(x) = ?$

4. Let $g(x) = \int_{1}^{\sin x} \sqrt{1+t^3}\, dt$. What is $g'(x) = ?$

5. Find $\frac{d}{dx}\left[\int_{x^2}^{3x} f(t)dt\right]$

6. Let $g(x) = \int_{5x}^{3x^2} \sqrt{1+t^3}\, dt$. What is $g'(x) = ?$

7. Evaluate

a. $\int_0^3 x^2 dx$

c. $\int_{-1}^{2} 3^x dx$

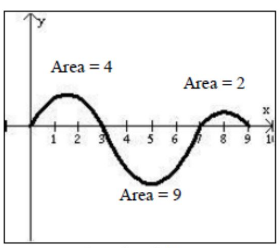
b. $\int_{\frac{\pi}{2}}^{\pi} (1+\cos x)dx$

d. $\int_4^9 f'(x)dx$

8. Given $\frac{dy}{dx} = 3x^2 + 4x - 5$ with the initial condition $y(2) = -1$, find $y(3)$

9. $f'(x) = \sin(x^2)$ and $f(2) = -5$. Find $f(1)$.

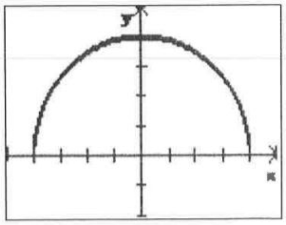
10. The graph of f' is shown at right, with areas of regions enclosed by the graph and the x-axis as indicated. Give that $f(3) = 5$, find $f(0)$, $f(7)$, and $f(9)$.



Graph of f'

11. A pizza with a temperature of $95^\circ C$ is put into a $25^\circ C$ room when $t = 0$. The pizza's temperature is decreasing at a rate of $r(t) = 6e^{-0.1t}\,^\circ C$ per minute. Estimate the pizza's temperature when $t = 5$ minutes.

12. The graph of f' is the semicircle shown at the right. Find $f(-4)$ if $f(4) = 7$



The Trapezoidal Rule

To approximate $\int_a^b f(x)\,dx$, use

$$T = \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n\right),$$

where $[a, b]$ is partitioned into n subintervals of equal length $h = (b - a)/n$.

Equivalently,

$$T = \frac{\text{LRAM}_n + \text{RRAM}_n}{2},$$

where LRAM_n and RRAM_n are the Riemann sums using the left and right endpoints, respectively, for f for the partition.

Simpson's Rule

To approximate $\int_a^b f(x)\,dx$, use

$$S = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right),$$

where $[a, b]$ is partitioned into an *even* number n of subintervals of equal length $h = (b - a)/n$.

5. Use the trapezoidal rule with $n = 4$ to approximate the value of $\int_0^2 (x^3 - x + 1)\,dx$.

6. The table below shows the velocity of a remote control car as it travelled down the hallway for 10 seconds. Using the trapezoidal rule, estimate the distance travelled by the car using 10 subintervals.

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (in./sec)	0	6	10	16	14	12	18	22	12	4	2

7. Use Simpson's Rule with $n = 4$ to approximate the value of $\int_1^2 \sin x\,dx$ and find the exact value of the integral to check your answer.