Rectangle: a parallelogram w/ 4 right angles

*b/c parallelogram keeps parallelogram properties

6.13	If a parallelogram is a rectangle then the diagonals are congruent	 $\overline{AC} \cong \overline{BD}$

If a quadrilateral is a rectangle the following properties are true:

r 1. All 4 angles are 90°

p 2. opposite sides parallel

p 3. opposite sides $\cong$

p 4. opposite angle $\cong$

p 5. consecutive angles supplementary

p 6. Diagonals bisect each other

r 7. Diagonals are $\cong$

1. Quadrilateral MNOP is a billboard in the shape of a rectangle. If $MO = 6x + 14$ and $PN = 9x + 5$, find x. Then find NR.

$6x + 14 = 9x + 5$

$9 = 3x$

$3 = x$

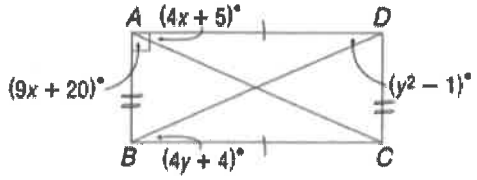
$NR = \frac{1}{2} PN$

$= \frac{1}{2}(9(3) + 5)$

$= \frac{1}{2}(32)$

$= 16$

2. Quadrilateral ABCD is a rectangle. Solve for the missing variables.



$4x + 5 + 9x + 20 = 90°$

$13x + 25 = 90$

$13x = 65$

$x = 5$

$\triangle ABD \cong \triangle CDB$

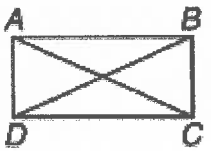
$4y + 4 = y^2 - 1$

$0 = y^2 - 4y - 5$

$0 = (y - 5)(y + 1)$

$y = 5, -1$ *$y = -1$ makes angles zero

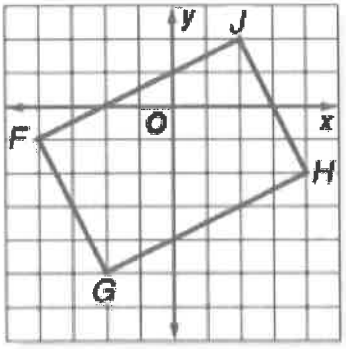
$y = 5$

6.14	If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.	

3. Quadrilateral FGHJ has vertices $F(-4,-1)$, $G(-2,-5)$, $H(4,-2)$, $J(2,2)$. Determine whether FGHJ is a rectangle.

* ≅ or ∥ would mean parallelogram not necessarily rectangle

Method 1: Show the opposite sides are perpendicular



$$m_{FG} = \frac{-1-(-5)}{-4-(-2)} = \frac{4}{-2} = -2$$

$$m_{FJ} = \frac{-1-2}{-4-2} = \frac{-3}{-6} = \frac{1}{2}$$

$\overline{FG} \perp \overline{FJ}$

$$m_{JH} = \frac{2-(-2)}{2-4} = \frac{4}{-2} = -2$$

$$m_{GH} = \frac{-5-(-2)}{-2-4} = \frac{-3}{-6} = \frac{1}{2}$$

$\overline{JH} \perp \overline{GH}$

Method 2: Show that the diagonals are congruent

$$d_{FH} = \sqrt{(-4-4)^2 + (-1-(-2))^2} = \sqrt{64+1} = \sqrt{65}$$

$$d_{GJ} = \sqrt{(-2-2)^2 + (-5-2)^2} = \sqrt{16+49} = \sqrt{65}$$

$\overline{FH} \cong \overline{GJ}$

4. Quadrilateral JKMN is a rectangle:

a. If $JQ = 5x - 3$ and $QM = 4x + 6$, find JM

$5x - 3 = 4x + 6$

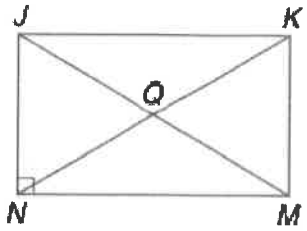
$x = 9$

$JM = 5x - 3 + 4x + 6$

$= 9x + 3$

$= 9(9) + 3$

$= 84$



b. If $NQ = 2x + 3$ and $QK = 5x - 9$, find NK

$2x + 3 = 5x - 9$

$12 = 3x$

$4 = x$

$NK = 2x + 3 + 5x - 9$

$= 7x - 6$

$= 7(4) - 6$

$= 22$

c. If $NM = x + 1$ and $JK = x^2 - 5$ find JK

$x + 1 = x^2 - 5$

$0 = x^2 - x - 6$

$0 = (x - 3)(x + 2)$

$x = 3, -2$

↑ makes NM neg

$x = 3$

$JK = x^2 - 5$

$= 3^2 - 5$

$= 4$

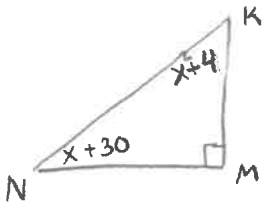
d. If $m\angle NJM = 2x - 3$ and $m\angle KJM = x + 5$, find x

diagonals bisect angles

$2x - 3 = x + 5$

$x = 8$

e. If $m\angle NKM = x^2+4$ and $m\angle KNM = x+30$ find $m\angle JKN$



$x^2+4 + x + 30 + 90 = 180$

$x^2 + x + 124 = 180$

$x^2 + x - 56 = 0$

$(x-7)(x+8) = 0$

$x = 7, -8$

$x = 7$

$m\angle JKN = m\angle KNM$

$= x + 30$

$= 7 + 30$

$= 37°$

f. If $m\angle JKN = 2x^2 + 2$ and $m\angle NKM = 14x$, find x.

44
1, 44
2, 22
4, 11

$2x^2 + 2 + 14x = 90$

$2x^2 + 14x - 88 = 0$

$2(x^2 + 7x - 44) = 0$

$(x+11)(x-4) = 0$

$x = -11, 4$

$x = 4$

5. WXYZ is a rectangle. Find each measure if $m\angle 1 = 30$.

a. $m\angle 2 = 60°$

b. $m\angle 3 = 60°$

c. $m\angle 4 = 30°$

d. $m\angle 5 = 30°$

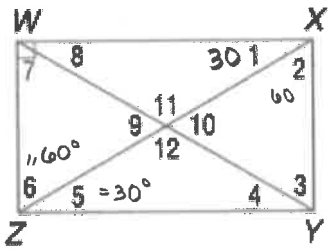
e. $m\angle 6 = 60°$

f. $m\angle 7 = 60°$

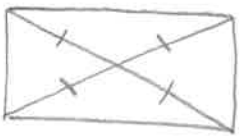
g. $m\angle 8 = 30°$

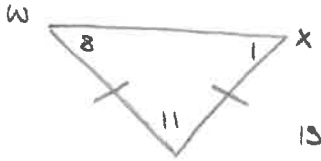
h. $m\angle 9 = 60°$

i. $m\angle 12 = 120°$



* diagonals are $\cong$ and bisect each other so





isosceles $\triangle$

$\Rightarrow$ base angles $\cong$