

Proof : $f(x) = \int_{-2}^x t dt$ $f'(x) = ?$

$$= \frac{1}{2}t^2 \Big|_{-2}^x = \frac{1}{2}x^2 - \frac{1}{2}(-2)^2$$

der of constant = 0

2019-20
AB Calculus

6.4 Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus (FTC)

Part 1

If f is continuous on $[a, b]$, then the function

$$F(x) = \int_a^x f(t) dt$$

has a derivative at every point x in $[a, b]$, and

$$\frac{dF}{dx} = \frac{d}{dx} \int_a^x f(t) dt = \underline{f(x)}$$

Part 2

If f is continuous on $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x) dx = \underline{F(b) - F(a)}$$

1. Find $\frac{dy}{dx}$

a. $y = \int_2^x (3t + \cot(t^2)) dt$

$$\frac{dy}{dx} = 3x + \cot(x^2)$$

c. $y = \int_6^{x^2} \cot 3t dt$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\text{der out}}{\text{inside}} \cot(3x^2) \frac{\text{der inside}}{\text{inside}} (2x) \\ &= 2x \cot(3x^2) \end{aligned}$$

b. $y = \int_4^x e^u \sec u du$

$$\frac{dy}{dx} = e^x \sec x$$

d. $y = \int_{3x^2}^{5x} \ln(2 + p^2) dp$

$$y = \int_0^{5x} \ln(2 + p^2) dp - \int_0^{3x^2} \ln(2 + p^2) dp$$

$$\frac{dy}{dx} = 5 \ln(2 + (5x)^2) - 6x \ln(2 + (3x^2)^2)$$

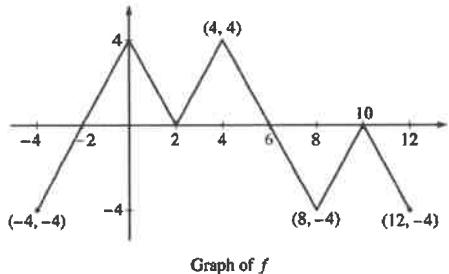
$$= 5 \ln(2 + 25x^2) - 6x \ln(2 + 9x^4)$$

$$= \ln \frac{(2 + 25x^2)^5}{(2 + 9x^4)^{6x}}$$

AB Calculus

6.4 Fundamental Theorem of Calculus

2. The figure below shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function g is defined by $g(x) = \int_{-4}^x f(t)dt$. Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.



$$g'(x) = f(x)$$

$$g'(x) = 0 \Rightarrow f(x) = 0 \quad x = -2, 2, 6, 10$$

$$\begin{array}{ccccccc} & \nearrow & + & \nearrow & \nearrow & \nearrow \\ -2 & & 2 & & 6 & & 10 \\ g'(x) = f(x) \end{array}$$

$g'(x)$ does not have a relative max or min at $x = 10$ b/c $f(x) \leq 0$ from $[8, 12]$

3. The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4+t^2} dt$ and $g(x) = f(\sin x)$. Find $f'(x)$ and $g'(x)$.

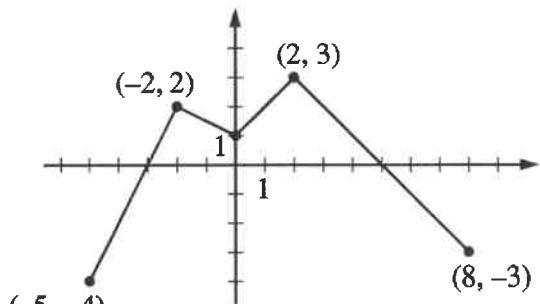
$$f'(x) = \sqrt{4 + (3x)^2} \quad (3)$$

$$g'(x) = f'(\sin x) \cos x$$

$$= 3\sqrt{4 + 9x^2}$$

$$= 3\cos x \sqrt{4 + 9\sin^2 x}$$

4. The continuous function f is defined on the interval $-5 \leq x \leq 8$. The graph of f which consists of four line segments is shown in the figure below. Let g be the function given by $g(x) = 2x + \int_{-2}^x f(t)dt$. Find $g'(x)$ in terms of $f(x)$. For each of $g''(4)$ and $g''(-2)$, find the value of state that it does not exist.



$$g'(x) = 2 + f(x)$$

$$g''(x) = f'(x)$$

$$g''(4) = f'(4) = \frac{-3 - 3}{8 - 2} = \frac{-6}{6} = -1$$

$$g''(-2) = f'(-2) = \text{DNE}$$

5. Evaluate each integral using the Evaluation Part of the Fundamental Theorem:

$$\text{a. } \int_2^{-1} 3^x dx = - \int_{-1}^2 3^x dx$$

$$= -\frac{3^x}{\ln 3} \Big|_{-1}^2$$

$$= -\left(\frac{9}{\ln 3} - \frac{1}{3\ln 3}\right)$$

$$\text{b. } \int_0^5 x^{\frac{3}{2}} dx = -\frac{26}{3\ln 3}$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} \Big|_0^5$$

$$= \frac{2}{5} (5)^{\frac{5}{2}} - 0$$

$$= \frac{2}{5} \sqrt{5}^5 = \frac{2}{5} 5^2 \sqrt{5} = 10\sqrt{5}$$

$$\text{c. } \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \csc^2 \theta d\theta$$

$$= -\cot \theta \Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= -\cot \frac{5\pi}{6} + \cot \frac{\pi}{6}$$

$$= \sqrt{3} + \sqrt{3}$$

$$= 2\sqrt{3}$$

$$\text{d. } \int_0^4 \frac{1-\sqrt{u}}{\sqrt{u}} du = \int_0^4 (u^{-\frac{1}{2}} - 1) du$$

$$= 2u^{\frac{1}{2}} - u \Big|_0^4$$

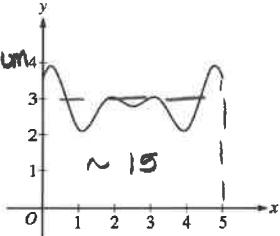
$$= (2\sqrt{4} - 4) - 0$$

$$= 0$$

6. The graph of f' , the derivative of f , is shown below. If $f(0) = 20$, which of the following could be the value of $f(5)$? initial condition

*to find area
could do

Riemann Sum
 f do one rectangle
area ~ 15

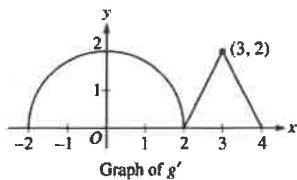


Graph of f'
some below and above rectangle

- A 15
B 20
C 25
D 35
E 40

$$\begin{aligned} \int_a^b f'(x) dx &= f(b) - f(a) \\ f(5) - f(0) &= \int_0^5 f'(x) dx \\ f(5) &= 20 + \int_0^5 f'(x) dx \\ &\approx 20 + 15 \end{aligned}$$

7. The graph of g' , the first derivative of g , consists of a semicircle of radius 2 and two line segments, as shown in the figure below. If $g(0) = 1$, what is $g(3)$?



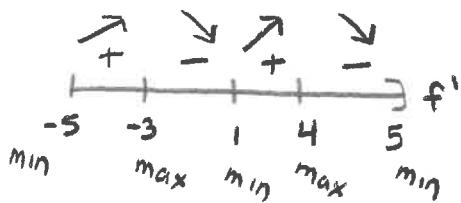
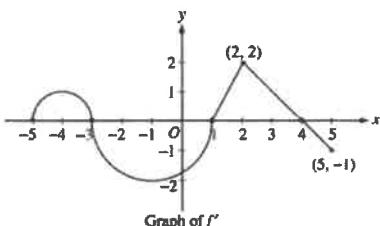
- A $\pi + 1$
B $\pi + 2$
C $2\pi + 1$
D $2\pi + 2$

$$\begin{aligned} g(3) - g(0) &= \int_0^3 g'(x) dx \\ g(3) &= 1 + \int_0^3 g'(x) dx \\ &= 1 + \left(\frac{1}{4}\pi(2)^2 + \frac{1}{2}(1)(2)\right) \\ &= 1 + \pi + 1 \\ &= \pi + 2 \end{aligned}$$

The absolute min of $f(x)$ over $-5 \leq x \leq 5$ is

3 at $x=1$ b/c $f'(x)$ changes from neg to pos at $x=1$ AB Calculus
and $f(1) < f(x)$ for $-5 \leq x \leq 5$. 6.4 Fundamental Theorem of Calculus

8. Let f be a function defined on the closed interval $-5 \leq x \leq 5$ with $f(1) = 3$. The graph of f' the derivative of f , consists of two semicircles and two line segments, as shown below. Find the absolute minimum value of $f(x)$ over the closed interval $-5 \leq x \leq 5$. Explain your reasoning.



$$f'(x) = 0 \text{ at } x = -5, -3, 1, 4$$

$f'(x)$ und? no

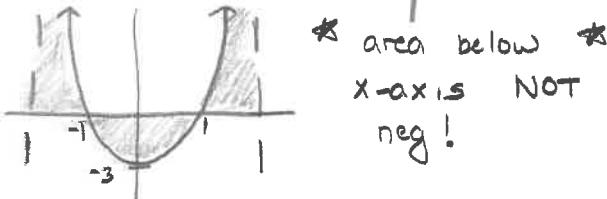
f' endpoints $x = -5, 5$

$$f(x) - f(1) = \int_1^x f'(t) dt$$

$$f(x) = 3 + \int_1^x f'(t) dt$$

x	$f(x)$
-5	$3 + \frac{3}{2}\pi$
1	3
5	4.5

9. Find the total area of the region bounded between the curve and the x-axis of $y = 3x^2 - 3$ on $-2 \leq x \leq 2$



$$\int_{-2}^2 |3x^2 - 3| dx = \left| \int_{-2}^{-1} (3x^2 - 3) dx \right| + \left| \int_{-1}^1 (3x^2 - 3) dx \right| + \left| \int_1^2 (3x^2 - 3) dx \right|$$

$$\int_{-2}^2 |3x^2 - 3| dx = \boxed{12}$$

#8 continued:

$$f(-5) = 3 + \int_{-5}^{-1} f'(t) dt$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^5 f'(t) dt$$

$$= 3 - \int_{-5}^{-1} f'(t) dt$$

$$= 3 + \frac{1}{2}(2)(2) - \frac{1}{2}(1)(1)$$

$$= 3 - \left(\frac{1}{2}\pi(1)^2 - \frac{1}{2}\pi(2)^2 \right)$$

$$= 3 + 2 - \frac{1}{2}$$

$$= 3 - (\pi/2 - 2\pi)$$

$$= 5 - \frac{1}{2}$$

$$= 3 + \frac{3}{2}\pi$$

$$= 4.5$$