DEFINITION Differential Equation

An equation involving a derivative is called a **differential equation.** The **order of a differential equation** is the order of the highest derivative involved in the equation.

1. Find the general solution of the exact differential equation:
   a. $\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x^2}\ (x > 0)$
   b. $\frac{dy}{dx} = \sec^2 x + 2x + 5$

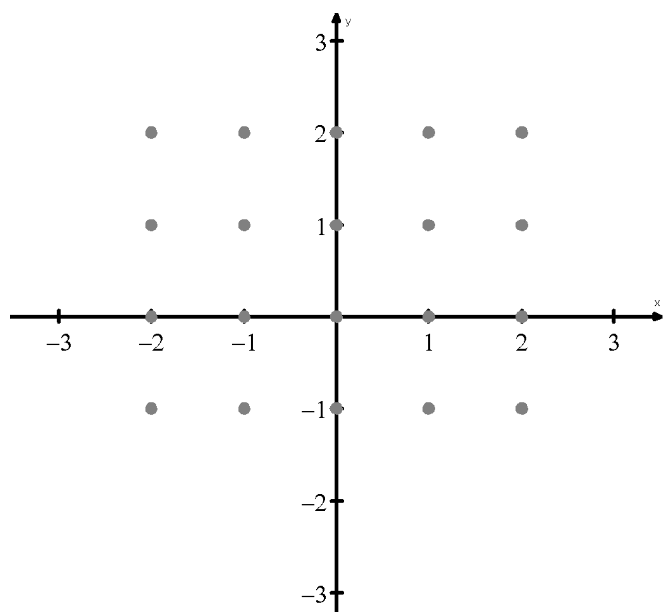
2. Find the particular solution to the equation $\frac{dy}{dx} = e^x - 6x^2$ whose graph passes through the point $(1, 0)$.

3. Find the particular solution to the differential equation $\frac{dy}{dx} = 2e^x - \cos x$ and $y = 3$ when $x = 0$.

4. Find $f(-2)$ with the differential equation $f'(x) = e^{-x^2}$ and $f(7) = 3$.

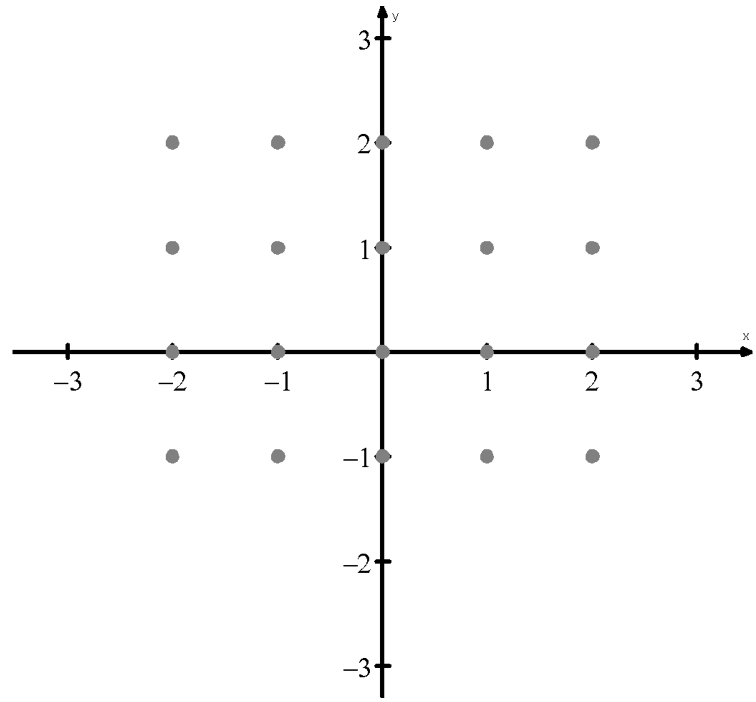
Not all differential equations can be solved using separation of variables. These are called inseparable differential equations. You can graphically represent the solution to a differential equation using a **slope field**. Slope fields use short line segments to represent the tangent lines of the solution to the differential equation at many different points. Together, the many segments give a fuller picture of what the solution would look like.

5. Use separation of variables to find the solution to the differential equation $\frac{dy}{dx} = x$. Then, graph the slope field for the differential equation at the points indicated on the graph.



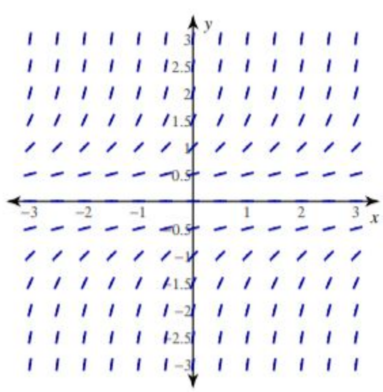
You can see the various solutions for the general solution to $\frac{dy}{dx} = x$ based on the value of C. You can also graph a particular solution on a slope field if you are given a point that the curve passes through. Begin at the initial condition and use the slope to sketch an approximate path for the solution curve.

6. Sketch a slope field for the differential equation y' = 2x + y at the indicated points. Sketch the solution that passes through (1, 1).



7. For each problem, find a differential equation that could be represented with the given slope field.

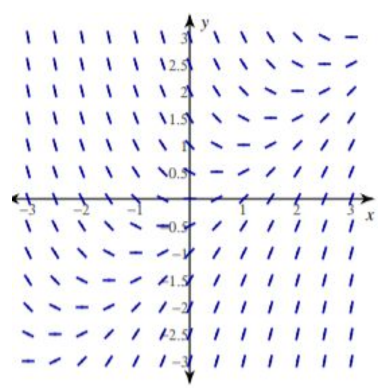
a.



A) $\frac{dy}{dx} = -\frac{1}{x}$ B) $\frac{dy}{dx} = -\frac{1}{y}$

C) $\frac{dy}{dx} = 1$ D) $\frac{dy}{dx} = y^2$

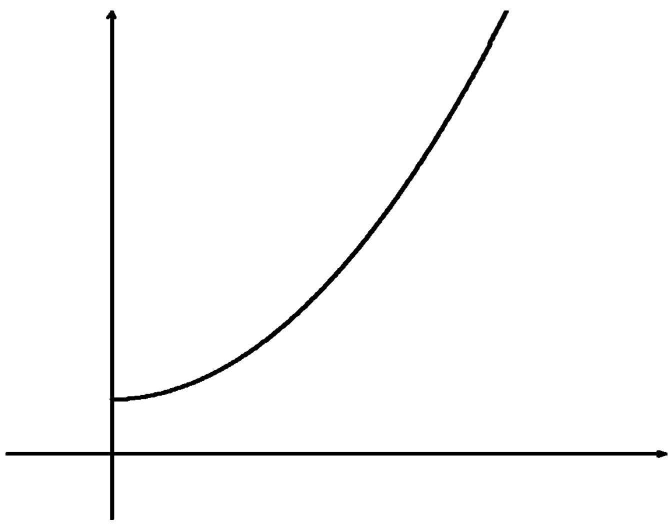
b.



A) $\frac{dy}{dx} = x + y$ B) $\frac{dy}{dx} = x - y$

C) $\frac{dy}{dx} = xy$ D) $\frac{dy}{dx} = -xy$

Euler's Method is a numerical approach to approximating the particular solution of a differential equation. Euler's Method uses the concept of local linearity to approximate the shape of the solution curve. Use the slope of the curve to create short, connected line segments.



- Use the slope at x_0 to create a tangent segment.
- The length of the segment is predetermined and called the step-size (h or Δx).
- The next segment (from x_1 to x_2) uses the slope at x_1 to find the next point x_2.
- This process continues until you reach the value that you are trying to find (i.e. approx. f(1.4)).

In general:
The smaller the step size, the better the approximation will be.

The approximation will be an overestimate if the curve is concave ______________ and an

underestimate if the curve is concave ______________.

8. Let $y = f(x)$ be the particular solution to the differential equation $y' = x - y$ with initial condition $f(0) = 1$. Use Euler's Method to approximate $f(1.5)$ using 5 equal step sizes.

9. Given $\frac{dy}{dx} = 2x + y$ and $y(1) = 3$, approximate y(0) using Euler's Method with 4 equal step sizes.

10. Given the differential equation $\frac{dy}{dx} = x + 2$ and $y(0) = 3$. Find an approximation for $y(1)$ by using Euler's method with two equal steps. Sketch your solution.

    a. Solve the differential equation $\frac{dy}{dx} = x + 2$ with the initial condition $y(0) = 3$, and use your solution to find $y(1)$.

b. The error in using Euler's Method is the difference between the approximate value and the exact value. What was the error in your answer? How could you produce a smaller error using Euler's Method?

11. Given the differential equation $\frac{dy}{dx} = \frac{1}{x+2}$ and $y(0) = 1$, find an approximation of $y(1)$ using Euler's Method with two steps and step size $\Delta x = 0.5$.

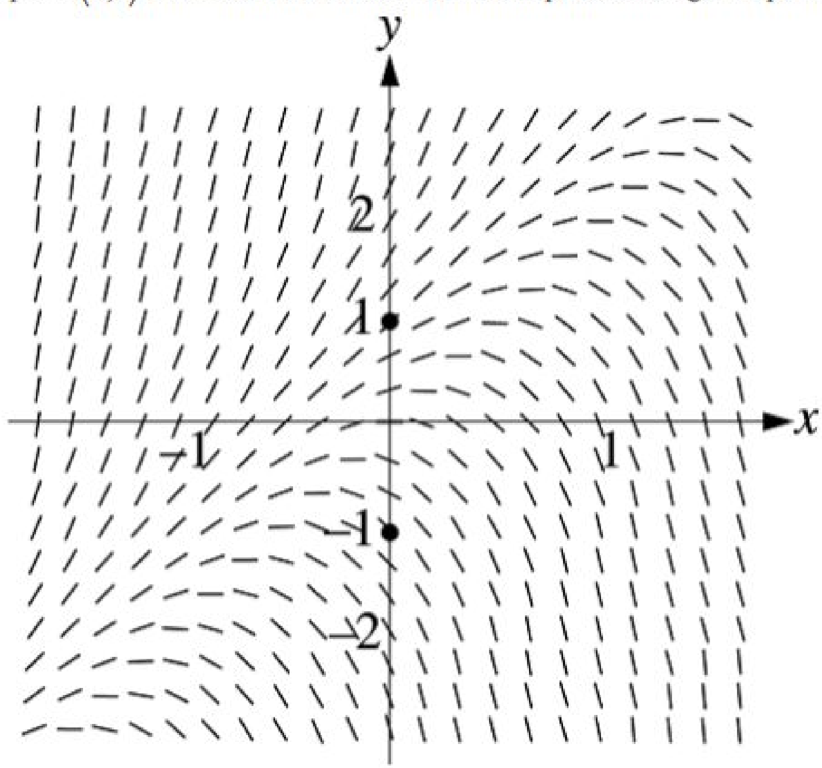
12. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = x + 2y$ with the initial condition $f(0) = 1$. Use Euler's Method, starting at $x = 0$ with two steps of equal size to approximate $f(-0.6)$.

13.

AP 2002-5 (No Calculator)

Consider the differential equation: $\frac{dy}{dx} = 2y - 4x$.

(a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0,1)$ and sketch the solution curve that passes through the point $(0,-1)$.



(b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

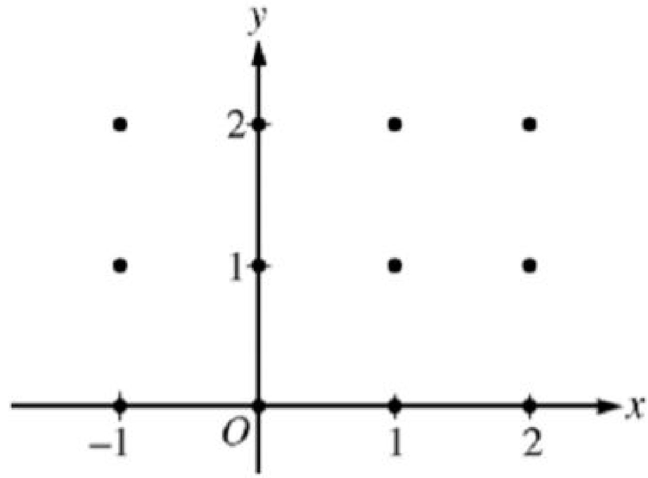
(c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

(d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0,0)$? If so, is the point a local maximum or a local minimum? Justify your answer.

14. AP 2005 - 4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated and sketch the solution curve that passes through the point $(0,1)$.



(b) The solution curve that passes through the point $(0,1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y-coordinate of this local minimum?

(c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.

(d) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.