

general solution: antiderivative

\* Don't forget your cupcake

AB Calculus

7.1 Slope Fields and Euler's Method

## DEFINITION Differential Equation

An equation involving a derivative is called a **differential equation**. The **order of a differential equation** is the order of the highest derivative involved in the equation.

1. Find the general solution of the exact differential equation:

a.  $\frac{dy}{dx} = \sec x \tan x - e^x$

$$dy = (\sec x \tan x - e^x) dx$$

$$\int dy = \int (\sec x \tan x - e^x) dx$$

$$y = \sec x - e^x + C$$

b.  $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{x}}$

$$dy = \left( \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{x}} \right) dx$$

$$\int dy = \int \left( \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{x}} \right) dx$$

$$y = \sin^{-1} x - 2\sqrt{x} + C$$

2. Solve the initial value problem explicitly:

a.  $\frac{dy}{dx} = 2e^x - \cos x$  and  $y = 3$

when  $x = 0$

$$\int dy = \int (2e^x - \cos x) dx$$

$$y = 2e^x - \sin x + C$$

$$3 = 2e^0 - \sin 0 + C$$

$$3 = 2 - 0 + C$$

$$1 = C$$

$$y = 2e^x - \sin x + 1$$

c.  $\frac{dy}{dt} = \cos t(e^{\sin t})$

$$dy = \cos t e^{\sin t} dt$$

$$\int dy = \int \cos t e^{\sin t} dt$$

$$u = \sin t$$
$$du = \cos t dt$$

$$\int dy = \int e^u du$$

$$y = e^{\sin t} + C$$

d.  $\frac{dy}{du} = 4(\sin u)^3 (\cos u)$

$$\int dy = \int 4(\sin u)^3 \cos u du$$

$$v = \sin u$$
$$dv = \cos u du$$

$$y = \int 4v^3 dv$$

$$y = (\sin u)^4 + C$$

b.  $\frac{dA}{dx} = 10x^9 + 5x^4 - 2x + 4$  and

$A = 6$  when  $x = 1$

$$\int dA = \int (10x^9 + 5x^4 - 2x + 4) dx$$

$$A = x^{10} + x^5 - x^2 + 4x + C$$

$$6 = 1^{10} + 1^5 - 1^2 + 4(1) + C$$

$$6 = 5 + C$$

$$1 = C$$

$$A = x^{10} + x^5 - x^2 + 4x + 1$$

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c.  $\frac{dy}{dx} = 5\sec^2 x - \frac{3}{2}\sqrt{x}$  and  
 $y = 7$  when  $x = 0$

$$\begin{aligned} \int dy &= \int (5\sec^2 x - \frac{3}{2}\sqrt{x}) dx \\ y &= 5\tan x - x^{3/2} + C \\ 7 &= 5\tan 0 - 0^{3/2} + C \\ 7 &= C \end{aligned}$$

$y = 5\tan x - x^{3/2} + 7$

d.  $\frac{ds}{dt} = t(3t - 2)$  and  $s = 0$   
when  $t = 1$

$$\begin{aligned} \int ds &= \int (3t^2 - 2t) dt \\ s &= t^3 - t^2 + C \\ 0 &= 1^3 - 1^2 + C \\ 0 &= C \end{aligned}$$

$s = t^3 - t^2$

3. Solve the initial value problem using the Fundamental Theorem (Your answer will contain a definite integral)

a.  $\frac{du}{dx} = \sqrt{2 + \cos x}$  and  $u = -3$  when  $x = 0$

$$u(x) = \int_0^x \sqrt{2 + \cos t} dt + C$$

$u(x) = \int_0^x \sqrt{2 + \cos t} dt - 3$

check:  $u(0) = \int_0^0 \sqrt{2 + \cos t} dt - 3 = -3 \checkmark$

a.  $G'(s) = \sqrt[3]{\tan s}$  and  $G(0) = 4$

$$G(s) = \int_0^s \sqrt[3]{\tan x} dx + 4$$

4. Draw a slope field for each of the following:

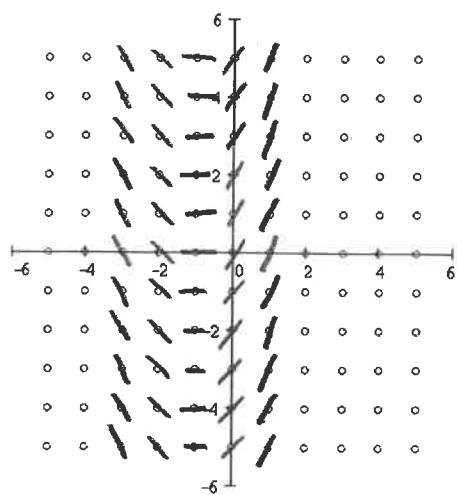
a.  $\frac{dy}{dx} = x + 1$

$x$	$y$	$\frac{dy}{dx}$
0	1	
-1	0	
-2	-1	

\* check

$$y = \int (x+1) dx$$

$$y = \frac{1}{2}x^2 + x + C$$



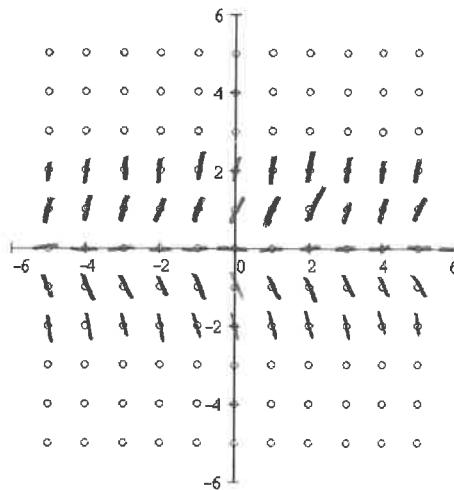
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b.  $\frac{dy}{dx} = 2y$

$$y=0 \quad \frac{dy}{dx} = 0$$

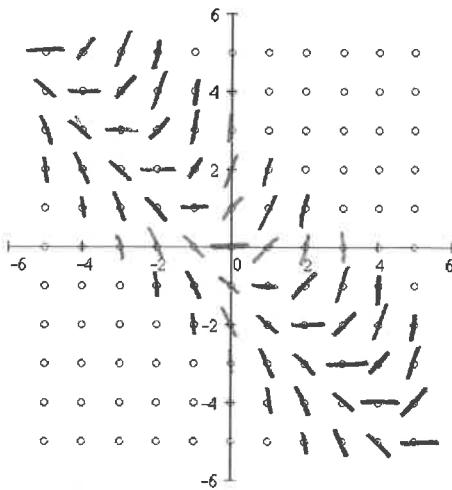
$$y=1 \quad \frac{dy}{dx} = 2$$

$$y=-1 \quad \frac{dy}{dx} = -2$$



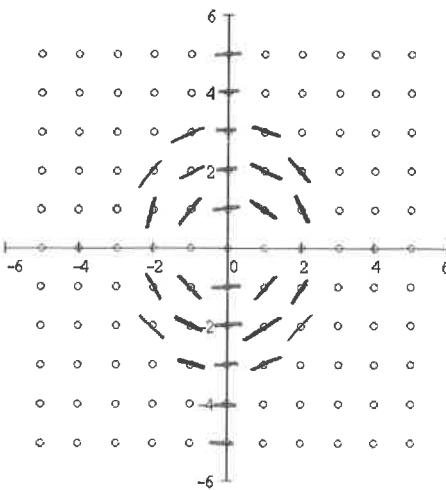
c.  $\frac{dy}{dx} = x + y$

x	y	$\frac{dy}{dx}$
0	0	0
1	1	2
2	1	3



d.  $\frac{dy}{dx} = -\frac{x}{y}$

x	y	$\frac{dy}{dx}$
0	0	und
0	1	0
1	1	-1
-1	1	1
1	2	-1/2



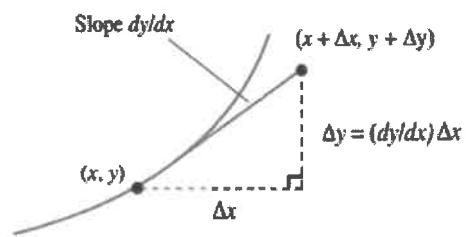
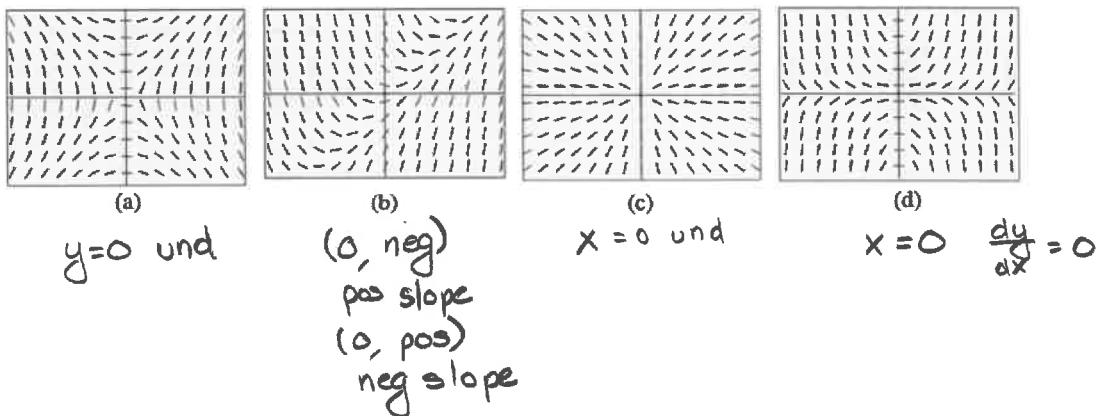
5.

Use slope analysis to match each of the following differential equations with one of the slope fields (a) through (d). (Do not use your graphing calculator.)

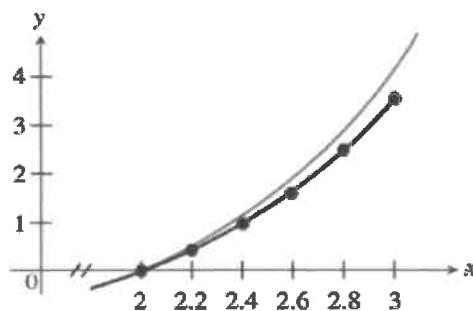
$$1. \frac{dy}{dx} = x - y \quad (b) \quad 2. \frac{dy}{dx} = xy \quad (d) \quad 3. \frac{dy}{dx} = \frac{x}{y} \quad (a) \quad 4. \frac{dy}{dx} = \frac{y}{x} \quad (c)$$

\* look for:

- 1) undefined
- 2) zero slope



**Figure 6.6** How Euler's Method moves along the linearization at the point  $(x, y)$  to define a new point  $(x + \Delta x, y + \Delta y)$ . The process is then repeated, starting with the new point.

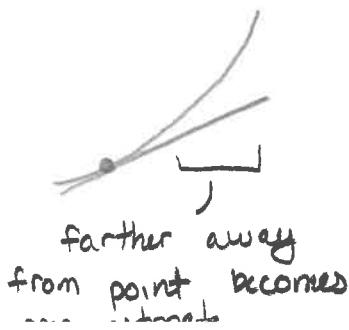


**Figure 6.7** Euler's Method is used to construct an approximate solution to an initial value problem between  $x = 2$  and  $x = 3$ . (Example 9)

approximate slope at 1 point

use new point to approximate next section of slope

distance between approximated slopes leads to a better approximation



$$*\frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

$$\Delta y = \frac{dy}{dx} \cdot \Delta x$$

### AB Calculus

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6. Use Euler's Method with increments  $\Delta x = 0.1$  to approximate the value of  $y$  when  $x = 1.3$

a.  $\frac{dy}{dx} = y - 1$  and  $y = 3$  when  $x = 1$

$(x, y)$	$\frac{dy}{dx}$	$\Delta x$	$\Delta y$	$(x + \Delta x, y + \Delta y)$
(1, 3)	2	0.1	0.2	(1.1, 3.2)
(1.1, 3.2)	2.2	0.1	0.22	(1.2, 3.42)
(1.2, 3.42)	2.42	0.1	0.242	(1.3, 3.662)

$$y \approx 3.662$$

b.  $\frac{dy}{dx} = 2x - y$  and  $y = 0$  when  $x = 1$

$(x, y)$	$\frac{dy}{dx}$	$\Delta x$	$\Delta y$	$(x + \Delta x, y + \Delta y)$
(1, 0)	2	0.1	0.2	(1.1, 0.2)
(1.1, 0.2)	2	0.1	0.2	(1.2, 0.4)
(1.2, 0.4)	2	0.1	0.2	(1.3, 0.6)

$$y \approx 0.6$$

7. Use Euler's Method with increments  $\Delta x = -0.1$  to approximate the value of  $y$  when  $x = 1.7$

a.  $\frac{dy}{dx} = 1 + y$  and  $y = 0$  when  $x = 2$

$(x, y)$	$\frac{dy}{dx}$	$\Delta x$	$\Delta y$	$(x + \Delta x, y + \Delta y)$
(2, 0)	1	-0.1	-0.1	(1.9, -0.1)
(1.9, -0.1)	0.9	-0.1	-0.09	(1.8, -0.19)
(1.8, -0.19)	0.81	-0.1	-0.081	(1.7, -0.271)

$$y \approx -0.271$$

b.  $\frac{dy}{dx} = x - 2y$  and  $y = 1$  when  $x = 2$

$(x, y)$	$\frac{dy}{dx}$	$\Delta x$	$\Delta y$	$(x + \Delta x, y + \Delta y)$
(2, 1)	0	-0.1	0	(1.9, 1)
(1.9, 1)	-0.1	-0.1	0.01	(1.8, 1.01)
(1.8, 1.01)	-0.22	-0.1	0.022	(1.7, 1.032)

$$y \approx 1.032$$

\*calc

euler( $\frac{dy}{dx}$ ,  $x$ ,  $y$ ,  $\{ \}$ ,  $y$  value,  $\Delta x$ )

