

DEFINITION Indefinite Integral

The family of all antiderivatives of a function $f(x)$ is the **indefinite integral of f with respect to x** and is denoted by $\int f(x)dx$.

If F is any function such that $F'(x) = f(x)$, then $\int f(x)dx = F(x) + C$, where C is an arbitrary constant, called the **constant of integration**.

Properties of Indefinite Integrals

$$\int k f(x)dx = k \int f(x) dx \quad \text{for any constant } k$$

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

Power Formulas

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \quad \text{when } n \neq -1$$

$$\int u^{-1} du = \int \frac{1}{u} du = \ln |u| + C$$

(see Example 2)

Trigonometric Formulas

$$\int \cos u du = \sin u + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

Exponential and Logarithmic Formulas

$$\int e^u du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int \ln u du = u \ln u - u + C \quad (\text{See Example 2})$$

$$\int \log_a u du = \int \frac{\ln u}{\ln a} du = \frac{u \ln u - u}{\ln a} + C$$

7.2 Antidifferentiation by Substitution

1. $\int 3x^2(x^3 + 5)^{20} dx$

3. $\int 10x(5x^2 - 3)^6 dx$

2. $\int (\cos x)^4 \sin x dx$

4. $\int x^4(x^5 - 12)^7 dx$

7.2 Antidifferentiation by Substitution

5. $\int x \sin x^2 dx$

7. $\int \frac{\sec^2 x}{\tan x} dx$

6. $\int 3 \sin(3x - 1) dx$

8. $\int x \cos(3x^2 + 1) dx$

7.2 Antidifferentiation by Substitution

9. $\int \frac{4x^6}{(x^7+8)^5} dx$

10. $\int x^3 \sqrt{x^2-6} dx$

Challenge!

11. $\int x \sqrt{x+9} dx$

7.2 Antidifferentiation by Substitution

12.
$$\int_1^2 2x(x^2 - 2)^3 dx$$

14.
$$\int_0^{\frac{1}{2}} (e^y + 2 \cos(\pi y)) dy$$

13.
$$\int_{-2}^0 2t^2 \sqrt{1 - 4t^3} dt$$

15.
$$\int_{-5}^5 \frac{4t}{2 - 8t^2} dt$$

$$16. \int_0^{\ln(1+\pi)} e^x \cos(1 - e^x) dx$$

Challenge!

$$17. \int_1^5 \frac{x}{\sqrt{2x-1}} dx$$