

DEFINITION Indefinite Integral

The family of all antiderivatives of a function $f(x)$ is the **indefinite integral of f with respect to x** and is denoted by $\int f(x)dx$.

If F is any function such that $F'(x) = f(x)$, then $\int f(x)dx = F(x) + C$, where C is an arbitrary constant, called the **constant of integration**.

Properties of Indefinite Integrals

$$\int k f(x)dx = k \int f(x) dx \quad \text{for any constant } k$$

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

Power Formulas

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \text{ when } n \neq -1$$

$$\int u^{-1} du = \int \frac{1}{u} du = \ln |u| + C$$

(see Example 2)

Trigonometric Formulas

$$\int \cos u du = \sin u + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

Exponential and Logarithmic Formulas

$$\int e^u du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int \ln u du = u \ln u - u + C \quad (\text{See Example 2})$$

$$\int \log_a u du = \int \frac{\ln u}{\ln a} du = \frac{u \ln u - u}{\ln a} + C$$

1. Evaluate indefinite integral:

a. $\int x^{-2} dx$

e. $\int 28(7x - 2)^3 dx$

b. $\int \frac{dt}{t^2 + 1}$

c. $\int (2e^x + \sec x \tan x - \sqrt{x}) dx$

d. $\int x \cos(2x^2) dx$

f. $\int \frac{9r^2 dr}{\sqrt{1-r^3}}$

BC Calculus

7.2 Antidifferentiation by Substitution

g. $\int 8(y^4 + 4y^2 + 1)^2(y^3 + 2y)dy$

j. $\int \sqrt{\cot x} \csc^2 x dx$

h. $\int \sec(\theta + \frac{\pi}{2}) \tan(\theta + \frac{\pi}{2}) d\theta$

k. $\int \tan^7(\frac{x}{2}) \sec^2(\frac{x}{2}) dx$

i. $\int 3(\sin x)^{-2} dx$

7.2 Antidifferentiation by Substitution

l. $\int \frac{dx}{\sin^2 3x}$

n. $\int \tan^2 x \sec^2 x dx$

m. $\int \frac{6 \cos t}{(2 + \sin t)^2} dt$

o. $\int \frac{40}{x^2 + 25} dx$

2. Make a u-substitution and integrate from $u(a)$ to $u(b)$

a. $\int_0^1 r\sqrt{1-r^2}dr$

c. $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4+3 \sin x}} dx$

b. $\int_{-1}^1 \frac{5r}{(4+r^2)^2} dr$

d. $\int_0^{\frac{\pi}{2}} \cos^{-3} 2\theta \sin 2\theta d\theta$

$$\text{e. } \int_2^5 \frac{dx}{2x-3}$$

$$\text{g. } \int_0^2 \frac{e^x dx}{3+e^x}$$

$$\text{f. } \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cot x dx$$

3. Use the given trigonometric identity to set up a u-substitution and then evaluate the indefinite integral.

a. $\int \sec^4 x dx, \sec^2 x = 1 + \tan^2 x$

b. $\int 4\cos^2 x dx, \cos 2x = 2\cos^2 x - 1$