Precalculus
5.1 Verifying Identities
Day 2

ex) $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

$\frac{\sin x\,(1-\cos x)}{(1+\cos x)(1-\cos x)}$

$\frac{\sin x\,(1-\cos x)}{1-\cos^2 x}$

$\frac{\sin x\,(1-\cos x)}{\sin^2 x}$

$\frac{1-\cos x}{\sin x}$

* need $1-\cos x$ in numerator

* don't foil if need term

ex) $\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = 2 + 2\cot^2\theta$

$\frac{1-\cos\theta + 1+\cos\theta}{(1+\cos\theta)(1-\cos\theta)}$

$\frac{2}{1-\cos^2\theta}$

$2 + 2\,\frac{\cos^2\theta}{\sin^2\theta}$

$2\,\frac{\sin^2\theta}{\sin^2\theta} + 2\,\frac{\cos^2\theta}{\sin^2\theta}$

$\frac{2\sin^2\theta + 2\cos^2\theta}{\sin^2\theta}$

$\frac{2(\sin^2\theta + \cos^2\theta)}{1-\cos^2\theta}$

$\frac{2}{1-\cos^2\theta}$

* need $1-\cos^2\theta$ in den

ex) $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

* many ways
  ↳ could use $\csc^2 x = 1 + \cot^2 x$

OR...

$$\frac{\csc^2 x}{\csc^2 x} - \frac{1}{\csc^2 x}$$

$$1 - \sin^2 x$$

$$\cos^2 x$$

ex) $(\csc x - \cot x)^2 = \dfrac{1-\cos x}{1+\cos x}$

$$\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)^2$$

$$\frac{(1-\cos x)^2}{\sin^2 x}$$

$$\frac{1-\cos x}{1+\cos x}\ \frac{(1-\cos x)}{(1-\cos x)}$$

$$\frac{(1-\cos x)^2}{1-\cos^2 x}$$

$$\frac{(1-\cos x)^2}{\sin^2 x}$$

ex) $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

*factor

$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$

$1(\sin^2 x - \cos^2 x)$

$\sin^2 x - \cos^2 x$

$1 - \cos^2 x - \cos^2 x$

$1 - 2\cos^2 x$

ex) $\dfrac{\sin x}{1-\cot x} - \dfrac{\cos x}{\tan x - 1} = \sin x + \cos x$

$$\frac{\sin x}{1 - \frac{\cos x}{\sin x}} - \frac{\cos x}{\frac{\sin x}{\cos x} - 1}$$

$$\frac{\sin x}{\sin x}\cdot\frac{\sin x}{1 - \frac{\cos x}{\sin x}} - \frac{\cos x}{\cos x}\cdot\frac{\cos x}{\frac{\sin x}{\cos x} - 1}$$

* eliminate fractions in denominator

$$\frac{\sin^2 x}{\sin x - \cos x} - \frac{\cos^2 x}{\sin x - \cos x}$$

$$\frac{\sin^2 x - \cos^2 x}{\sin x - \cos x}$$

$$\frac{(\sin x + \cos x)(\sin x - \cos x)}{\sin x - \cos x}$$

$$\sin x + \cos x$$