Double Angle Identities:

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

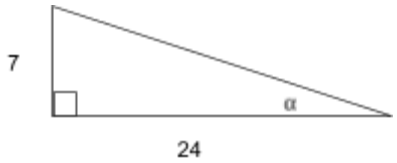
$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

1. Use the figure to find the exact value of:
   a. $\sin 2\alpha$

   b. $\cos 2\alpha$



   c. $\tan 2\alpha$

2. If $\cot\theta = 3$, and θ lies in quadrant III, find $\cos 2\theta$

3. If $\sin\theta = \frac{15}{17}$, and θ lies in quadrant II, find $\tan 2\theta$.

4. Write each expression as the sine, cosine, or tangent of a double angle. Then find the exact value of the expression.

   a. $1 - 2\sin^2 \frac{\pi}{12}$

   c. $2 \sin 22.5^\circ \cos 22.5^\circ$

   b. $\cos^2 105^\circ - \sin^2 105^\circ$

   d. $\frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$

5. Verify each identity:

   a. $\sin 2\theta = \frac{2 \cot \theta}{1 + \cot^2 \theta}$

   b. $(\sin \theta - \cos \theta)^2 = 1 - \sin 2\theta$

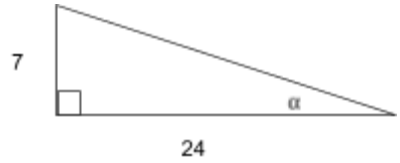
Half Angle Identities:

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

6. Use the figure to find the exact value of:
   a. $\sin\frac{\alpha}{2}$



   b. $\cos\frac{\alpha}{2}$

   c. $\tan\frac{\alpha}{2}$

7. Use a half angle formula to find the exact value of each expression.
   a. $\cos 22.5^\circ$

   b. $\sin 105^\circ$

   c. $\tan 112.5^\circ$

   d. $\tan\frac{3\pi}{8}$

8. $\tan\alpha = \frac{8}{15}$, α lies in quadrant III. Find:
   a. $\sin\frac{\alpha}{2}$

   b. $\cos\frac{\alpha}{2}$

   c. $\tan\frac{\alpha}{2}$

9. Verify the identity: $\tan\frac{\alpha}{2} = \frac{\tan\alpha}{\sec\alpha+1}$