Recall that if a function is defined so that ***each range element is used only once,*** then it is a
___________________-___________________-___________________ **function.**

Recall that the **inverse function** of the one-to-one function f is defined as follows.

$$f^{-1} = \{(y, x) \mid (x, y) \text{ belongs to } f\}$$

Do not confuse the −1 in f^{-1} with a negative exponent. The symbol $f^{-1}(x)$ represents the
____________ ____________ of f, *not* $\frac{1}{f(x)}$.

Review of Inverse Functions

1. In a one-to-one function, each x-value corresponds to ____________ ____________ y-value and each y-value corresponds to ____________ ____________ x-value.

2. If a function f is one-to-one, then f has an ____________ ____________ f^{-1}.

3. The domain of f is the ____________ of f^{-1}, and the range of f is the ____________ f^{-1}. That is, if the point (a, b) is on the graph of f, then the point ____________ lies on the graph of f^{-1}.

4. The graphs of f and f^{-1} are ____________ of each other across the line $y = x$.

5. To find $f^{-1}(x)$ from $f(x)$, follow these steps.

Step 1 ____

Step 2 ____

Step 3 ____

Inverse Sine Function

Inverse Sine Function

$y = \sin^{-1} x$ or $y = \arcsin x$ means that $x = \sin y$, for $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

We can think of $y = \sin^{-1} x$ ***or*** $y = \arcsin x$ ***as***

"y is the number (angle) in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ***whose sine is x."***

The domain of $y = \sin^{-1} x$ is ______________. The range of $y = \sin^{-1} x$ is ______________.

CLASSROOM EXAMPLE 1 Finding Inverse Sine Values

Find the value of each real number y if it exists.

(a) $y = \arcsin \dfrac{\sqrt{3}}{2}$

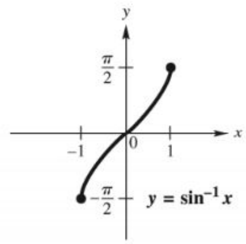
(b) $y = \sin^{-1}\left(-\dfrac{1}{2}\right)$

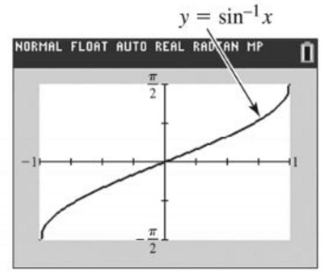
(c) $y = \sin^{-1} \sqrt{2}$

Be certain that the number given for an inverse function value is in the range of the particular inverse function being considered.

Inverse Sine Function $y = \sin^{-1} x$ **or** $y = \arcsin x$

x	y
-1	$-\frac{\pi}{2}$
$-\frac{\sqrt{2}}{2}$	$-\frac{\pi}{4}$
0	0
$\frac{\sqrt{2}}{2}$	$\frac{\pi}{4}$
1	$\frac{\pi}{2}$





Domain: __________

Range: __________

- The inverse sine function is increasing on the open interval __________ and continuous on its domain __________.
- Its x- and y-intercepts are both __________.
- Its graph is symmetric with respect to the __________, so the function is an __________ function. For all x in the domain, $\sin^{-1}(-x) =$ __________.

Inverse Cosine Function

Inverse Cosine Function

$$y = \cos^{-1} x \text{ or } y = \arccos x \text{ means that } x = \cos y, \text{ for } 0 \le y \le \pi.$$

We can think of $y = \cos^{-1} x$ ***or*** $y = \arccos x$ ***as***

"y is the number (angle) in the interval $[0, \pi]$ ***whose cosine is x."***

The domain of $y = \cos^{-1} x$ is _____________. The range of $y = \cos^{-1} x$ is _____________.

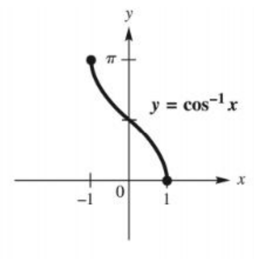
CLASSROOM EXAMPLE 2 Finding Inverse Cosine Values

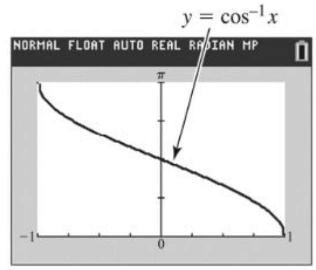
Find the value of each real number y if it exists.

(a) $y = \arccos 0$ **(b)** $y = \cot^{-1} \frac{1}{2}$

Inverse Cosine Function $y = \cos^{-1} x$ **or** $y = \arccos x$

x	y
-1	π
$-\frac{\sqrt{2}}{2}$	$\frac{3\pi}{4}$
0	$\frac{\pi}{2}$
$\frac{\sqrt{2}}{2}$	$\frac{\pi}{4}$
1	0





Domain: __________

Range: __________

- The inverse cosine function is decreasing on the open interval __________ and continuous on its domain __________.
- Its x-intercept is __________ and is y-intercept is __________.
- Its graph is not symmetric with respect to either the __________ or the __________.

<u>Inverse Tangent Function</u>

Inverse Tangent Function

$y = \tan^{-1} x$ or $y = \arctan x$ means that $x = \tan y$, for $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

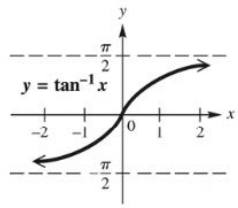
We can think of $y = \tan^{-1} x$ ***or*** $y = \arctan x$ ***as***

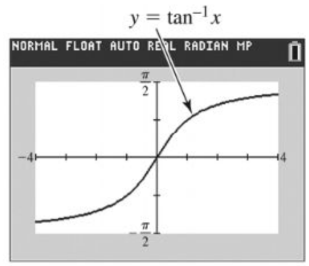
"y is the number (angle) in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ ***whose tangent is x."***

The domain of $y = \tan^{-1} x$ is ____________. The range of $y = \tan^{-1} x$ is ____________.

Inverse Tangent Function $y = \tan^{-1} x$ **or** $y = \arctan x$

x	y
-1	$-\frac{\pi}{4}$
$-\frac{\sqrt{3}}{3}$	$-\frac{\pi}{6}$
0	0
$\frac{\sqrt{3}}{3}$	$\frac{\pi}{6}$
1	$\frac{\pi}{4}$





Domain: __________

Range: __________

- The inverse tangent function is increasing on __________ and continuous on its domain __________.
- Its x- and y-intercepts are both __________.
- Its graph is symmetric with respect to the __________, so the function is an __________ function. For all x in the domain, $\tan^{-1}(-x) =$ __________.
- The lines __________ and __________ are horizontal asymptotes.

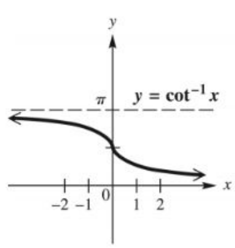
Other Inverse Circular Functions

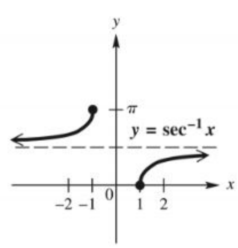
Inverse Cotangent, Secant, and Cosecant Functions

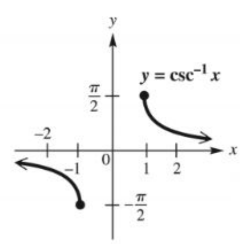
$y = \cot^{-1} x$ or $y = \text{arccot}\, x$ means that $x = \cot y$, for $0 < y < \pi$.

$y = \sec^{-1} x$ or $y = \text{arcsec}\, x$ means that $x = \sec y$, for $0 \le y \le \pi$, $y \ne \frac{\pi}{2}$.

$y = \csc^{-1} x$ or $y = \text{arccsc}\, x$ means that $x = \csc y$, for $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, $y \ne 0$.







Summary of Inverse Circular Functions

Inverse Function	Domain	Range	
		Interval	Quadrants of the Unit Circle
$y = \sin^{-1} x$			
$y = \cos^{-1} x$			
$y = \tan^{-1} x$			
$y = \cot^{-1} x$			
$y = \sec^{-1} x$			
$y = \csc^{-1} x$			

Inverse Function Values

CLASSROOM EXAMPLE 3 **Finding Inverse Function Values (Degree-Measured Angles)**

Find the *degree measure* of θ if it exists.

(a) $\theta = \arctan\sqrt{3}$ **(b)** $\theta = \csc^{-1}\left(-\sqrt{2}\right)$

Use the following to evaluate these inverse trigonometric functions on a calculator.

$\sec^{-1} x$ ***is evaluated as*** $\cos^{-1}\frac{1}{x}$; $\csc^{-1} x$ ***is evaluated as*** $\sin^{-1}\frac{1}{x}$;

$\cot^{-1} x$ ***is evaluated as*** $\begin{cases} \tan^{-1}\frac{1}{x} & \text{if } x > 0 \\ 180^\circ + \tan^{-1}\frac{1}{x} & \text{if } x < 0. \end{cases}$ Degree mode

CLASSROOM EXAMPLE 4 Finding Inverse Function Values with a Calculator

Use a calculator to approximate each value.

(a) Find y in radians if $y = \sec^{-1}(-4)$.

(b) Find θ in degrees if $\theta = \text{arccot}(-0.2528)$.

Be careful when using a calculator to evaluate the inverse cotangent of a negative quantity. Enter the inverse tangent of the _____________ of the negative quantity, which returns an angle in quadrant _____________. Because inverse cotangent is _____________ in quadrant II, adjust the calculator result by adding 180° or π accordingly. (Note that $\cot^{-1} 0 = \frac{\pi}{2}$ or 90°.)

CLASSROOM EXAMPLE 5 Finding Function Values Using Definitions of the Trigonometric Functions

Evaluate each expression without using a calculator.

(a) $\cos\left(\sin^{-1}\frac{2}{3}\right)$ **(b)** $\sec\left(\cot^{-1}\left(-\frac{15}{8}\right)\right)$

CLASSROOM EXAMPLE 6 Finding Function Values Using Identities

Evaluate each expression without using a calculator.

(a) $\sin\left(\arctan\frac{4}{3}-\arccos\frac{12}{13}\right)$ **(b)** $\sin\left(2\operatorname{arccot}(-5)\right)$