Recall that if a function is defined so that ***each range element is used only once,*** then it is a
one - to - one **function.**

Recall that the **inverse function** of the one-to-one function f is defined as follows.

$$f^{-1} = \{(y, x) \mid (x, y) \text{ belongs to } f\}$$

Do not confuse the −1 in f^{-1} with a negative exponent. The symbol $f^{-1}(x)$ represents the
inverse function of f, *not* $\frac{1}{f(x)}$.

Review of Inverse Functions

1. In a one-to-one function, each x-value corresponds to only one y-value and each y-value corresponds to only one x-value.
2. If a function f is one-to-one, then f has an inverse function f^{-1}.
3. The domain of f is the range of f^{-1}, and the range of f is the domain f^{-1}. That is, if the point (a, b) is on the graph of f, then the point (b, a) lies on the graph of f^{-1}.
4. The graphs of f and f^{-1} are reflections of each other across the line $y = x$.
5. To find $f^{-1}(x)$ from $f(x)$, follow these steps.

Step 1 Replace f(x) w/ y and interchange x and y

Step 2 solve for y

Step 3 Replace y w/ $f^{-1}(x)$

*what angle gives ___

Inverse Sine Function

Inverse Sine Function
$y = \sin^{-1} x$ or $y = \arcsin x$ means that $x = \sin y$, for $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

We can think of $y = \sin^{-1} x$ *or* $y = \arcsin x$ *as*
"y is the number (angle) in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ *whose sine is x."*

exists here

The domain of $y = \sin^{-1} x$ is $[-1, 1]$. The range of $y = \sin^{-1} x$ is $[-\pi/2, \pi/2]$.

CLASSROOM EXAMPLE 1 Finding Inverse Sine Values
Find the value of each real number y if it exists.

1 60 2
30
$\sqrt{3}$

(a) $y = \arcsin \frac{\sqrt{3}}{2}$

A C

$\theta' = 60°$

$\boxed{y = \pi/3}$

(b) $y = \sin^{-1}\left(-\frac{1}{2}\right)$

A C

$30 = \pi/6$

$\theta' = 30°$

$\theta = \frac{11\pi}{6}$ ← to get to $\frac{11\pi}{6}$ needs to revolve an undefined

$\boxed{\theta = -\pi/6}$

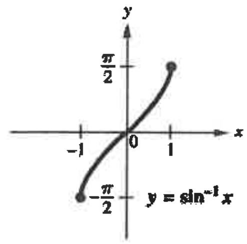
(c) $y = \sin^{-1} \sqrt{2}$

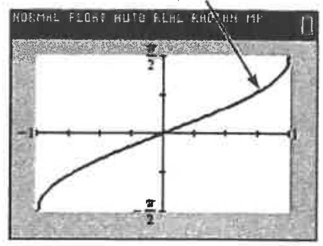
$\boxed{\text{No solution}}$

Be certain that the number given for an inverse function value is in the range of the particular inverse function being considered.

Inverse Sine Function $y = \sin^{-1} x$ **or** $y = \arcsin x$

x	y
-1	$-\frac{\pi}{2}$
$-\frac{\sqrt{2}}{2}$	$-\frac{\pi}{4}$
0	0
$\frac{\sqrt{2}}{2}$	$\frac{\pi}{4}$
1	$\frac{\pi}{2}$





Domain: $[-1, 1]$
Range: $[-\pi/2, \pi/2]$

- The inverse sine function is increasing on the open interval $[-1, 1]$ and continuous on its domain $[-1, 1]$.
- Its x- and y-intercepts are both $(0, 0)$.
- Its graph is symmetric with respect to the origin, so the function is an odd function. For all x in the domain, $\sin^{-1}(-x) =$ $-\sin^{-1}(x)$

Inverse Cosine Function

Inverse Cosine Function

$y = \cos^{-1} x$ or $y = \arccos x$ means that $x = \cos y$, for $0 \le y \le \pi$.

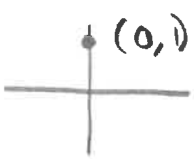
We can think of $y = \cos^{-1} x$ ***or*** $y = \arccos x$ ***as***

"y is the number (angle) in the interval $[0, \pi]$ ***whose cosine is x."***

The domain of $y = \cos^{-1} x$ is $[-1, 1]$. The range of $y = \cos^{-1} x$ is $[0, \pi]$.

CLASSROOM EXAMPLE 2 Finding Inverse Cosine Values

Find the value of each real number y if it exists.



(a) $y = \arccos 0$

$\boxed{y = \pi/2}$

(b) $y = \cos^{-1} \frac{1}{2}$

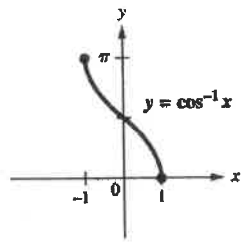
S | A

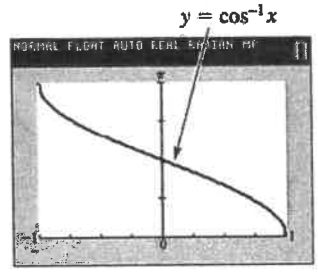
$\theta' = 60°$

$\boxed{y = \pi/3}$

Inverse Cosine Function $y = \cos^{-1} x$ or $y = \arccos x$

x	y
-1	π
$-\frac{\sqrt{2}}{2}$	$\frac{3\pi}{4}$
0	$\frac{\pi}{2}$
$\frac{\sqrt{2}}{2}$	$\frac{\pi}{4}$
1	0





Domain: $[-1, 1]$

Range: $[0, \pi]$

- The inverse cosine function is decreasing on the open interval $[-1, 1]$ and continuous on its domain $[-1, 1]$.
- Its x-intercept is $(1, 0)$ and is y-intercept is $(0, \pi/2)$.
- Its graph is not symmetric with respect to either the origin or the y-axis.

Inverse Tangent Function

Inverse Tangent Function

$y = \tan^{-1} x$ or $y = \arctan x$ means that $x = \tan y$, for $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

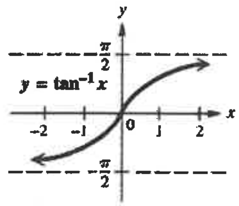
We can think of $y = \tan^{-1} x$ ***or*** $y = \arctan x$ ***as***

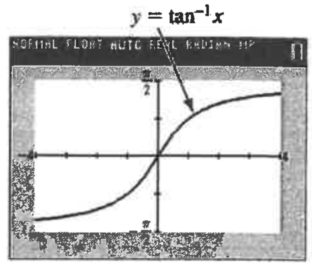
"y is the number (angle) in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ ***whose tangent is x."***

The domain of $y = \tan^{-1} x$ is $(-\infty, \infty)$. The range of $y = \tan^{-1} x$ is $(-\pi/2, \pi/2)$

Inverse Tangent Function $y = \tan^{-1} x$ or $y = \arctan x$

x	y
-1	$-\frac{\pi}{4}$
$-\frac{\sqrt{3}}{3}$	$-\frac{\pi}{6}$
0	0
$\frac{\sqrt{3}}{3}$	$\frac{\pi}{6}$
1	$\frac{\pi}{4}$





Domain: $(-\infty, \infty)$

Range: $(-\pi/2, \pi/2)$

- The inverse tangent function is increasing on $(-\infty, \infty)$ and continuous on its domain $(-\infty, \infty)$.
- Its x- and y-intercepts are both $(0,0)$.
- Its graph is symmetric with respect to the origin, so the function is an odd function. For all x in the domain, $\tan^{-1}(-x) = -\tan^{-1}(x)$
- The lines __________ and __________ are horizontal asymptotes.

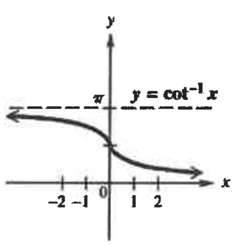
Other Inverse Circular Functions

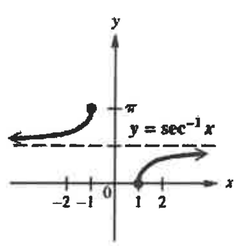
Inverse Cotangent, Secant, and Cosecant Functions

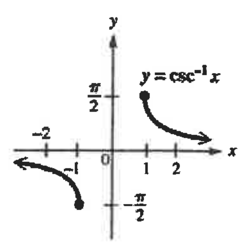
$y = \cot^{-1} x$ or $y = \text{arccot}\, x$ means that $x = \cot y$, for $0 < y < \pi$.

$y = \sec^{-1} x$ or $y = \text{arcsec}\, x$ means that $x = \sec y$, for $0 \le y \le \pi$, $y \ne \frac{\pi}{2}$.

$y = \csc^{-1} x$ or $y = \text{arccsc}\, x$ means that $x = \csc y$, for $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, $y \ne 0$.







Summary of Inverse Circular Functions

Inverse Function	Domain	Range	
		Interval	Quadrants of the Unit Circle
$y = \sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$	I & IV
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$	I & II
$y = \tan^{-1} x$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$	I & IV
$y = \cot^{-1} x$	$(-\infty, \infty)$	$(0, \pi)$	I & II
$y = \sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi]$	I & II
$y = \csc^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[-\pi/2, \pi/2]$	I & IV

Inverse Function Values

CLASSROOM EXAMPLE 3 Finding Inverse Function Values (Degree-Measured Angles)

Find the *degree measure* of θ if it exists.

(a) $\theta = \arctan\sqrt{3}$

A
C

$\theta' = 60°$

$\boxed{\theta = \pi/3}$

(b) $\theta = \csc^{-1}\left(-\sqrt{2}\right)$

A
C

$\theta' = 45°$

$\boxed{\theta = -\pi/4}$

Use the following to evaluate these inverse trigonometric functions on a calculator.

$\sec^{-1} x$ *is evaluated as* $\cos^{-1} \frac{1}{x}$; $\csc^{-1} x$ *is evaluated as* $\sin^{-1} \frac{1}{x}$;

$$\cot^{-1} x \text{ is evaluated as } \begin{cases} \tan^{-1} \frac{1}{x} & \text{if } x > 0 \\ 180° + \tan^{-1} \frac{1}{x} & \text{if } x < 0. \end{cases} \quad \text{Degree mode}$$

don't need
have inspire

CLASSROOM EXAMPLE 4 Finding Inverse Function Values with a Calculator
Use a calculator to approximate each value.

(a) Find y in radians if $y = \sec^{-1}(-4)$.

$= 1.823$

(b) Find θ in degrees if $\theta = \text{arccot}(-0.2528)$.

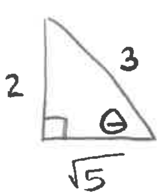
$= 1.818$

Be careful when using a calculator to evaluate the inverse cotangent of a negative quantity. Enter the inverse tangent of the ____________ of the negative quantity, which returns an angle in quadrant ____________. Because inverse cotangent is ____________ in quadrant II, adjust the calculator result by adding 180° or π accordingly. (Note that $\cot^{-1} 0 = \frac{\pi}{2}$ or 90°.)

dont need
have inspire

CLASSROOM EXAMPLE 5 Finding Function Values Using Definitions of the Trigonometric Functions
Evaluate each expression without using a calculator.

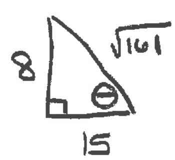
(a) $\cos\left(\sin^{-1} \frac{2}{3}\right)$ **(b)** $\sec\left(\cot^{-1}\left(-\frac{15}{8}\right)\right)$



$\sin^{-1} 2/3 = \theta$

$= \cos\theta$

$= \sqrt{5}/3$



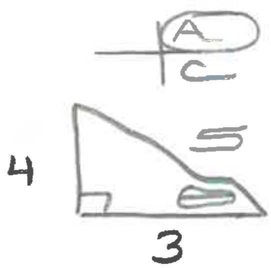
$= \sec\theta$

$= -\frac{\sqrt{161}}{15}$

CLASSROOM EXAMPLE 6 Finding Function Values Using Identities

Evaluate each expression without using a calculator.

(a) $\sin\left(\arctan\frac{4}{3} - \arccos\frac{12}{13}\right)$



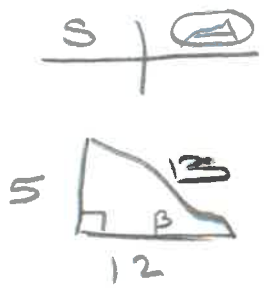
$\sin(\theta - \beta)$

$= \sin\theta\cos\beta - \cos\theta\sin\beta$

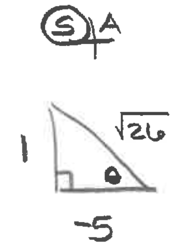
$= \frac{4}{5}\left(\frac{12}{13}\right) - \frac{3}{5}\left(\frac{5}{13}\right)$

$= \frac{48}{65} - \frac{15}{65}$

$= \frac{33}{65}$



(b) $\sin(2\operatorname{arccot}(-5))$



$= \sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\frac{1}{\sqrt{26}}\right)\left(\frac{-5}{\sqrt{26}}\right)$

$= \frac{-10}{26}$

$= -\frac{5}{13}$