Why Partial Fractions:

$\frac{3}{x-4} - \frac{2}{x+2} =$

The degree of the numerator must be SMALLER than the degree of the denominator

Table 7.2

S.No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$,
	where $x^2 + bx + c$ cannot be factorised further	

Steps:

1. Set up with unknown constants; A, B, C,.. in numerator
2.
3.
4.
5.
6.

1. Finding a Partial Fraction Decomposition:
   a. $\frac{x+14}{(x-4)(x+2)}$

   b. $\frac{x-18}{x(x-3)^2}$

c. $\frac{3x^2+17x+14}{(x-2)(x^2+2x+4)}$

2. Evaluate the Integrals:

   a. $\int \frac{3x^4+1}{x^2-1}\,dx$

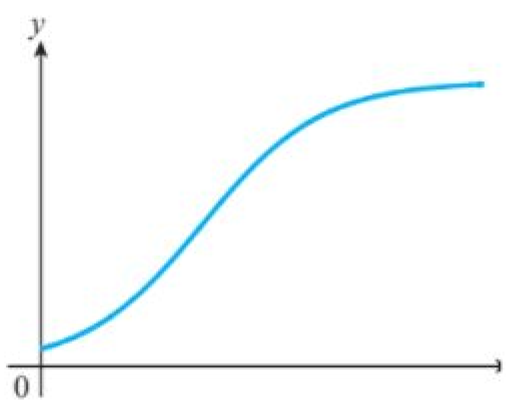
   b. $\int \frac{2x+16}{x^2+x-6}\,dx$

c. $\int \frac{3}{x^2+9} dx$

3. Find the integral without using the technique of partial fractions:

   a. $\int \frac{4x-3}{2x^2-3x+1} dx$

Logistic Curve



Can be modeled by $\frac{dP}{dt} = kp$ for some $k > 0$

If we want the growth rate to approach 0 as P approaches a maximal **carrying capacity** M**,** then:

$$\frac{dP}{dt} = kp(M - P)$$

4. The growth rate of a population P of bears in a newly established wildlife preserve is modeled by the differential equation $\frac{dP}{dt} = 0.008P(100 - P)$, where t is measured in years.
   a. What is the carrying capacity for bears in this wildlife preserve?

   b. What is the bear population when the population is growing the fastest?

   c. What is the rate of change of the population when it is growing the fastest?

5. In 1985 and 1987, the Michigan Department of Natural REsources airlifted 61 moose from Algonquin Park, Ontario, to Marquette County in the Upper Peninsula. It was originally hoped that the population P would reach carrying capacity in about 25 years with a growth rate of
$$\frac{dP}{dt} = 0.0003P(1000 - P).$$
   a. According to the model, what is the carrying capacity?

   b. With a calculator, generate a slope field for the differential equation.

The General Logistic Formula

The solution of the general logistic differential equation

$$\frac{dP}{dt} = kP(M - P)$$

is

$$P = \frac{M}{1 + Ae^{-(Mk)t}}$$

where A is a constant determined by an appropriate initial condition. The **carrying capacity** M and the **growth constant** k are positive constants.

6. The growth of the population of Aurora, CO, for the years between 1950 and 2003 was roughly logistic, satisfying the differential equation $\frac{dP}{dt} = P(0.1 - 3.125 \times 10^{-7}P$. Model the growth with a logistic function, using the initial condition $P(0) = 12,800$.