Finding position from displacement:

$$s(a) - s(o) = \int_0^a v(t)dt$$

$$s(a) = s(0) + \int_0^a v(t)dt$$

1. For the problems below, the function $v(t)$ is the velocity in *m/sec* of a particle moving along the x-axis. Use analytic methods to do each of the following:
   a. Determine when the particle is moving to the right, to the left, and stopped.
   b. Find the particle's displacement for the given time interval. If $s(0) = 3$, what is the particle's final position?
   c. Find the total distance traveled.
      i. $v(t) = 5\cos t, \quad 0 \le t \le 2\pi$

(a) $0 = 5\cos t$

$t = \pi/2,\ 3\pi/2$

$+ \quad \pi/2 \quad - \quad 3\pi/2 \quad + \quad v$

right $(0, \pi/2) \cup (3\pi/2, 2\pi)$

left $(\pi/2, 3\pi/2)$

stops $t = \pi/2,\ 3\pi/2$

b) displacement $= \int_0^{2\pi} 5\cos t\,dt$

$= 5\sin t \Big|_0^{2\pi}$

$= 5\sin 2\pi - 5\sin 0$

$\boxed{= 0}$

$s(2\pi) = s(0) + \int_0^{2\pi} 5\cos t\,dt$

$= 3 + 0$

$\boxed{= 3}$ ← particles final position

c)

Total distance $= \left|\int_0^{\pi/2} 5\cos t\,dt\right| + \left|\int_{\pi/2}^{3\pi/2} 5\cos t\,dt\right| + \left|\int_{3\pi/2}^{2\pi} 5\cos t\,dt\right|$

$= \left|5\sin t \Big|_0^{\pi/2}\right| + \left|5\sin t \Big|_{\pi/2}^{3\pi/2}\right| + \left|5\sin t \Big|_{3\pi/2}^{2\pi}\right|$

$= \left|5\sin \pi/2 - 5\sin 0\right| + \left|\underset{-5}{5\sin 3\pi/2} - \underset{-5}{5\sin \pi/2}\right| + \left|5\sin 2\pi - 5\sin 3\pi/2\right|$ $-(-5)$

$= 5 + 10 + 5$ $\boxed{= 20}$

2. For the problems below, the function $v(t)$ is the velocity in *m/sec* of a particle moving along the x-axis. Use analytic methods to do each of the following:
   a. Find the particle's displacement for the given time interval. If $s(0) = 3$, what is the particle's final position?
   b. Find the total distance traveled.
      i. $v(t) = 49 - 9.8t, \quad 0 \le t \le 10$

a) $\text{displacement} = \int_0^{10} (49 - 9.8t)\,dt$

$= 49t - \frac{9.8}{2}t^2 \Big|_0^{10}$

$= \left(490 - \frac{980}{2}\right) - (0)$

$\boxed{= 0}$

$s(10) = s(0) + \int_0^{10} (49 - 9.8t)\,dt$

$s(10) = 3 + 0 = \boxed{3}$

b) $0 = 49 - 9.8t$

$t = 5$

$\text{total distance} = \left|\int_0^{5} v(t)\,dt\right| + \left|\int_5^{10} v(t)\,dt\right|$

$= \left|49t - 4.9t^2 \Big|_0^5\right| +$

$\left|49t - 4.9t^2 \Big|_5^{10}\right|$

$= 122.5 + 122.5$

$\boxed{= 245}$

      ii. $v(t) = \sqrt{4 - t}, \quad 0 \le t \le 4$

a) $\text{disp} = \int_0^4 \sqrt{4-t}\,dt$

$= -\frac{2}{3}(4-t)^{3/2} \Big|_0^4$

$= -\frac{2}{3}(4-4)^{3/2} + \frac{2}{3}(4)^{3/2}$

$= \frac{16}{3}$

$s(4) = s(0) + \int_0^4 \sqrt{4-t}\,dt$

$= 3 + \frac{16}{3}$

$\boxed{= \frac{25}{3}}$

b) $0 = \sqrt{4-t}$

$t = 4$

$\text{total distance} = \int_0^4 v(t)\,dt$

$= \boxed{\frac{16}{3}}$

3. An automobile accelerates from rest at $1 + 3\sqrt{t}$ mph/sec for 9 seconds. $v(0) = 0$

   a. What is its velocity after 9 seconds? $a(t) = 1 + 3\sqrt{t}$

$$v(t) = \int a(t)\,dt$$
$$= t + 3\tfrac{2}{3}t^{3/2} + C$$
$$= t + 2t^{3/2} + C$$
$$0 = 0 + 2(0)^{3/2} + C \qquad C = 0$$

$$v(t) = t + 2t^{3/2}$$
$$v(9) = 9 + 2(9)^{3/2}$$
$$= 9 + 2(27)$$
$$\boxed{= 63 \text{ mph}}$$

   b. How far does it travel in those 9 seconds?

* check units

$\int_0^9 v(t)\,dt$

$\frac{mi}{hr} \cdot sec$

want miles

$\int_0^9 \frac{v(t)}{3600}\,dt$

$\frac{mi}{hr} \cdot sec \cdot \frac{1\ hr}{3600\ sec}$

$$\int_0^9 \frac{v(t)}{3600}\,dt$$
$$= \frac{1}{3600}\int_0^9 \left(t + 2t^{3/2}\right)dt$$
$$= \frac{1}{3600}\left(\frac{t^2}{2} + 2\,\frac{2t^{5/2}}{5}\right)\Bigg|_0^9$$
$$= \frac{1}{3600}\left(\frac{9^2}{2} + \frac{4}{5}(9^{5/2}) - (0)\right)$$
$$= \frac{1}{3600}\left(2349/10\right)$$
$$= 0.06525 \text{ mi} = \boxed{344.52 \text{ ft}}$$

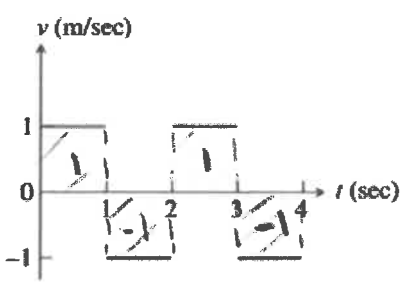
multiply by 5280 to convert

4. The graph of the velocity of a particle moving on the x-axis is given. The particle starts at $x = 2$ when $t = 0$. $s(0) = 2$

   a. Find where the particle is at the end of the trip.

   b. Find the total distance traveled by the particle.

i.



a) $s(4) = ?$

$$s(4) = s(0) + \int_0^4 v(t)\,dt$$
$$s(4) = 2 + 0$$
$$\boxed{= 2}$$

b) Total distance $\boxed{= 4\text{m}}$

Areas

① $\frac{1}{2}(2)(3) = 3$

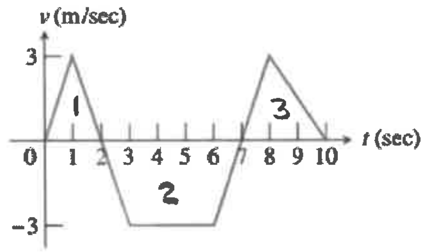
② $\frac{h}{2}(b_1 + b_2)$

$= -\frac{3}{2}(5+3)$

$= -12$

③ $\frac{1}{2}(3)(3) = \frac{9}{2}$

ii.



displacement $= \int_0^{10} v(t)\,dt$

$= 3 + (-12) + \frac{9}{2}$

$= -\frac{9}{2}$

a) $s(10) = s(0) + \int_0^{10} v(t)\,dt$

$s(10) = 2 + (-9/2)$

$\boxed{= -5/2}$

b) Total distance $=$

$3 + |-12| + 9/2$

$= \frac{39}{2} = \boxed{19.5\text{m}}$

4. The rate at which your home consumes electricity is measured in kilowatts. If your home consumes electricity at the rate of 1 kilowatt for 1 hour, you will be charged for 1 "kilowatt-hour" of electricity. Suppose that the average consumption rate for a certain home is modeled by the function $C(t) = 3.9 - 2.4\sin(\frac{\pi t}{12})$, where $C(t)$ is measured in kilowatts and t is the number of hours past midnight. Find the average daily consumption of this home measured in kilowatt-hours. units good!

↓
24 hrs

$= \int_0^{24} [3.9 - 2.4\sin(\pi t/12)]\,dt$

$u = \frac{\pi t}{12}$

$du = \pi/12\,dt$

$\frac{12}{\pi}du = dt$

$= 3.9t - 2.4 \cdot \frac{12}{\pi}\cos \pi t/12 \Big|_0^{24}$

$= \left(3.9(24) - \frac{28.8}{\pi}\cos\frac{24\pi}{12}\right) - \left(0 - \frac{28.8}{\pi}\cos 0\right)$

$= \boxed{93.6 \text{ Kilowatt-hours}}$

5. Midday traffic through an intersection can be modeled by the function $74 + 6\cos(\frac{t}{3})$ cars per minutes, where t is measured in minutes after noon. Find the number of cars that pass through this intersection between noon and 12:30 p.m.

★ measured in minutes ★

$$\int_0^{30} (74 + 6\cos(t/3))\, dt$$

$$= \left(74t + 6(3)\sin(t/3)\right)\Big|_0^{30}$$

$$= \left(74(30) + 18\sin(30/3)\right) - \left(74(0) + 18\sin(0/3)\right)$$

$$= 2220 - 9.792 - (0)$$

$$\approx \boxed{2210 \text{ cars}}$$