

2019-20
8.1 Accumulation and Net Change
AB Calculus

*includes HWK

2019 AB

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15 \sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

*store eqs in *
calc

- (a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)? Give your answer to the nearest whole number.
- (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)?
- (c) At what time t , for $0 \leq t \leq 8$, is the greatest number of fish in the lake? Justify your answer.
- (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$)? Explain your reasoning.

max \Rightarrow der = 0

a) $\int_0^5 E(t) dt = 153.45769$ 153 fish in the lake from midnight to 5 am

b) $\frac{1}{5-0} \cdot \int_0^5 L(t) dt = 6.0590$

c) let $A(t)$ represent the total number of fish in the lake at time t .

$$A(t) = \int_0^t [E(x) - L(x)] dx + A(0) \quad * \quad A(t) - A(0) = \int_0^t [E(x) - L(x)] dx$$

* $A(0) = 0$

$$A'(t) = E(t) - L(t)$$

$$0 = A'(t) \text{ when } t = 6.2035643$$

and check endpoints $t = 0, 8$

The greatest number of fish in the lake occurs @ time $t = 6.2035$ when 135.0149 fish are in the lake

t	$A(t)$
0	$\int_0^0 [E(x) - L(x)] dx = 0$
6.203564	$\int_0^{6.203564} [E(x) - L(x)] dx = 135.0149$
8	$\int_0^8 [E(x) - L(x)] dx = 80.9199$

2019 AB

t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	-29	55	48

2. The velocity of a particle, P , moving along the x -axis is given by the differentiable function v_P , where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.

~~(a) Justify why there must be at least one time t , for $0.3 \leq t \leq 2.8$, at which $v_P'(t)$, the acceleration of particle P , equals 0 meters per hour per hour.~~

~~(b) Use a trapezoidal sum with the three subintervals $[0, 0.3]$, $[0.3, 1.7]$, and $[1.7, 2.8]$ to approximate the value of $\int_0^{2.8} v_P(t) dt$.~~

- (c) A second particle, Q , also moves along the x -axis so that its velocity for $0 \leq t \leq 4$ is given by

$$v_Q(t) = 45\sqrt{t} \cos(0.063t^2) \text{ meters per hour. Find the time interval during which the velocity of particle } Q$$

is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the

velocity of particle Q is at least 60 meters per hour.
 do NOT need to say "total!"
 distance traveled = absolute value

- (d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.

distance traveled \neq displacement

c) $v_Q(t) \geq 60$ * graph $v_Q(t)$ and $y=60$

find intersection points & interval where

$$v_Q(t) \geq 60$$

$$1.86618 \leq t \leq 3.51917$$

$$3.5191744$$

$$\int_{1.86618147}^{3.5191744} |v_Q(t)| dt = 106.1087$$

$$1.86618147$$

* make sure decimal settings are "float"

can solve w/ interval

$$\textcircled{1} \text{ nsolve}(60 = v_Q(x), x, \overset{\text{lower bound}}{0}, \overset{\text{upper bound}}{4})$$

$$\textcircled{2} \text{ solve}(60 = v_Q(x), x | 0 \leq x \leq 4)$$

be careful \rightarrow numerical solve only gives answer closest to origin so adjust interval for each t

$$\text{nsolve}(60 = v_Q(t), x, 2, 4)$$

$$d) x_Q(2.8) - x_Q(0) = \int_0^{2.8} v_Q(t) dt$$

$$x_Q(2.8) = -90 + \int_0^{2.8} v_Q(t) dt$$

$$\approx 45.937653$$

$$x_P(2.8) - x_P(0) = \int_0^{2.8} v_P(t) dt$$

(continued on next page)

2019 #1
d) * rate of change inc/dec \Rightarrow der inc/dec \Rightarrow second der *
pos or neg

$$E'(5) - L'(5) \approx -10.72278 < 0$$

Since $E'(5) - L'(5) < 0$ the rate of change in
the number of fish is decreasing @ time $t=5$

2019 #2

d continued)

$$x_p(2.8) = 0 + \int_0^{2.8} v_p(t) dt$$

$$\approx \frac{1}{2}(0.3)(0+55) + \frac{1}{2}(1.4)(55+(-29)) + \frac{1}{2}(1.1)(-29+55)$$

$$= 40.75$$

At time $t=2.8$ the distance between particles P and
Q is approximately $45.937653 - 40.75 = 5.1876$ meters

2018 #1

d continued)

$$A(t) = 20 + \int_0^t (r(x) - 0.7) dx$$

$$A'(t) = r(t) - 0.7$$

$$0 = r(t) - 0.7$$

$$t = 33.0132978, 166.57471928$$

check endpoints $t=0, 300$

(work on back)

t	$A(t)$
0	20
33.0132978	$20 + \int_0^{33.0132978} (r(t) - 0.7) dt \approx 3.8034$
166.57471928	$20 + \int_0^{166.57471928} (r(t) - 0.7) dt \approx 158.0701$
300	80

At time $t = 33.0132$ the number of people in line is a minimum during $0 \leq t \leq 300$ with 4 people in line

2018 AB

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \leq t \leq 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

- (a) How many people enter the line for the escalator during the time interval $0 \leq t \leq 300$? *initial condition*
- (b) During the time interval $0 \leq t \leq 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?
- (c) For $t > 300$, what is the first time t that there are no people in line for the escalator?
- (d) For $0 \leq t \leq 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

$$a) \int_0^{300} r(t) dt = 270$$

b) Let $A(t)$ represent the amount of people in line at time t

$$A(t) = 20 + \int_0^t (r(x) - 0.7) dx$$

$$* A(t) - A(0) = \int_0^t (r(x) - 0.7) dx$$

$$A(300) = 20 + \int_0^{300} (r(x) - 0.7) dx$$

$$= 80$$

$$c) A(t) = 0 \quad t > 300$$

$$A(t) = 80 + \int_{300}^t -0.7 dx$$

* $r(t)$ only $0 \leq t \leq 300$

$$0 = 80 + \int_{300}^t -0.7 dx$$

$$t = 414, 2857$$

2018 AB

2. A particle moves along the x -axis with velocity given by $v(t) = \frac{10 \sin(0.4t^2)}{t^2 - t + 3}$ for time $0 \leq t \leq 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

~~(a) Find the acceleration of the particle at time $t = 3$.~~

(b) Find the position of the particle at time $t = 3$.

(c) Evaluate $\int_0^{3.5} v(t) dt$, and evaluate $\int_0^{3.5} |v(t)| dt$. Interpret the meaning of each integral in the context of the problem.

(d) A second particle moves along the x -axis with position given by $x_2(t) = t^2 - t$ for $0 \leq t \leq 3.5$. At what time t are the two particles moving with the same velocity?

$$b) \quad x(3) - x(0) = \int_0^3 v(t) dt$$

$$x(3) = -5 + \int_0^3 v(t) dt$$

$$x(3) \approx -1.7602$$

$$c) \quad \int_0^{3.5} v(t) dt \approx 2.8439$$

The particle's displacement from time 0 to time 3.5 is 2.8439.

$$\int_0^{3.5} |v(t)| dt \approx 3.737$$

The particle's total distance traveled from time 0 to time 3.5 is 3.737.

$$d) \quad v_2(t) = 2t - 1$$

$$v_2(t) = v(t)$$

$$t = 1.57054$$

The particle's are moving with the same velocity at time $t = 1.5705$

2017 AB

initial condition

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t) \sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \leq 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4 \ln(t^2 + 2t) \text{ for } 3 < t \leq 12,$$

≡ doesn't start @ 0.

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

- (a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?
- (b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.
- (c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.
- (d) How many pounds of bananas are on the display table at time $t = 8$?

a) $\int_0^2 f(t) dt \approx 20.05117$

c) $g(5) \approx 11.5328$

$f(5) \approx 13.7959$

b) $f'(7) \approx -8.1195$

since $f(5) > g(5)$ bananas are leaving the display table more quickly than they are arriving and the number of bananas is decreasing at time $t=5$.

The rate at which bananas are being removed from the shelf is decreasing by 8.1195 lbs per hour per hour at time $t=7$.

*if say -8.1195 can't say decreasing

- d) let $A(t)$ represent amount of bananas on display.

$$A(t) = A(0) + \int_3^t g(t) dt - \int_0^t f(t) dt$$

$$A(8) = 50 + \int_3^8 g(t) dt - \int_0^8 f(t) dt$$

$A(8) \approx 23.34739$

2016 AB

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.
 - (a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.
 - (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
 - (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
 - (d) For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

2015 AB

1. The rate at which rainwater flows into a drainpipe is modeled by the function R , where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.
- (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$?
 - (b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.
 - (c) At what time t , $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.
 - (d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

2014 AB

1. Grass clippings are placed in a bin, where they decompose. For $0 \leq t \leq 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.
- (a) Find the average rate of change of $A(t)$ over the interval $0 \leq t \leq 30$. Indicate units of measure.
 - (b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.
 - (c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \leq t \leq 30$.
 - (d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.