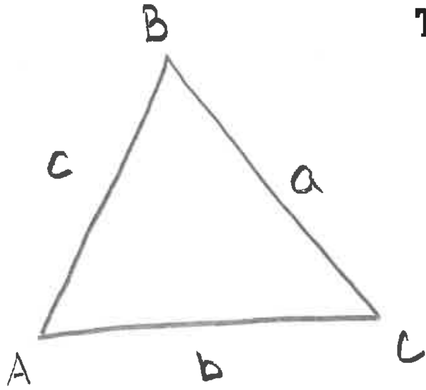
The Law of Sines

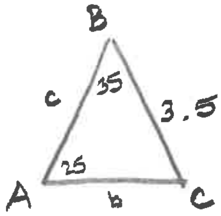


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

★BOB Rule!
Biggest side opposite biggest angle

1. Solve each triangle. Round side lengths and angle measures to the nearest tenth.

a. $a = 3.5,\ A = 25°,\ B = 35°$



$C = 180 - (25 + 35)$

$$\frac{b}{\sin 35} = \frac{3.5}{\sin 25}$$

$b = 4.8$

$$\frac{c}{\sin 120} = \frac{3.5}{\sin 25}$$

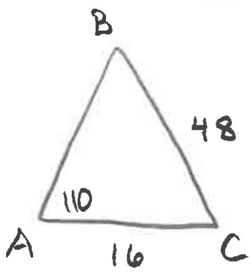
$c = 7.2$

$A = 25°$ $\quad a = 3.5$

$B = 35°$ $\quad b = \underline{4.8}$

$C = \underline{120°}$ $\quad c = \underline{7.2}$

b. $a = 48,\ A = 110°,\ b = 16$



$A = 110°$ $\quad a = 48$

$B = \underline{18.3°}$ $\quad b = 16$

$C = \underline{51.7°}$ $\quad c = \underline{40.1}$

$$\frac{48}{\sin 110} = \frac{16}{\sin B}$$

$$\frac{16 \sin 110}{48} = \sin B$$

$$\sin^{-1}\left(\frac{16 \sin 110}{48}\right) = B$$

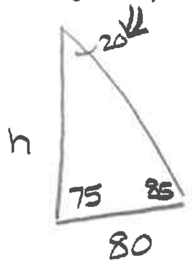
$B = 18.3°$

$$\frac{48}{\sin 110} = \frac{c}{\sin 51.7}$$

$40.1 = c$

2. A pine tree growing on a hillside makes a 75° angle with the hill. From a point [illegible] feet up the hill, the angle of elevation to the top of the tree is 62° and the angle of depression to the bottom is 23°. Find, to the nearest foot, the height of the tree.

180 -(75+85)

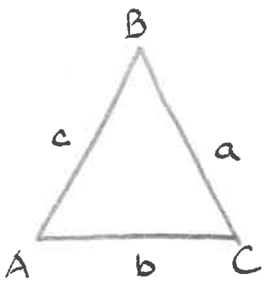


$$\frac{h}{\sin 85} = \frac{80}{\sin 20}$$

$h = 233.0$ ft

$\boxed{\text{height} = 233 \text{ ft}}$

The Law of Cosines



$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

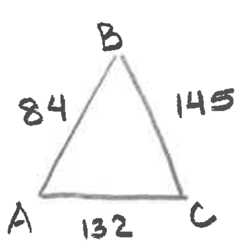
3. Solve each triangle. Round side lengths and angle measures to the nearest tenth.

   a. $a = 145,\ b = 132,\ c = 84$

$132^2 = 145^2 + 84^2 - (2)(145)(84) \cos B$

$0.437479 = \cos B$

$64.1^\circ = B$



A = $\underline{81^\circ}$ a = 145

B = $\underline{64.1^\circ}$ b = 132

C = $\underline{34.9^\circ}$ c = 84

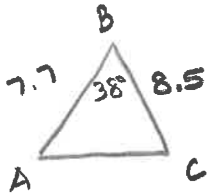
* could have used law of sines to get 2nd angle

$145^2 = 132^2 + 84^2 - 2(132)(84) \cos A$

$0.155799062 = \cos A$

$A = 81.0^\circ$

b. $a = 8.5,\ c = 7.7,\ B = 38°$



$b^2 = 8.5^2 + 7.7^2 - 2(8.5)(7.7)\cos 38$

$b = 5.3$

* store full decimal to use

$\frac{5.3}{\sin 38} = \frac{8.5}{\sin A}$

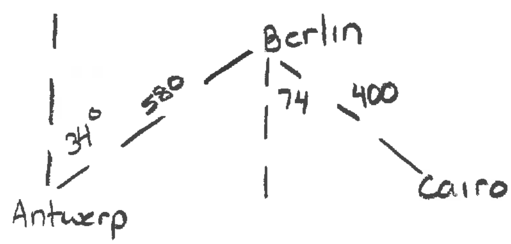
$\frac{8.5 \sin 38}{5.3} = \sin A \qquad 79.2° = A$

A = $\underline{79.2°}$ a = 8.5

B = 38° b = $\underline{5.3}$

C = $\underline{62.8°}$ c = 7.7

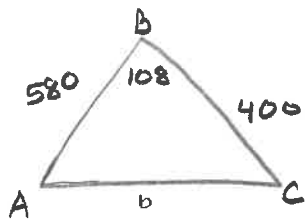
4. A plane leaves an airport in Antwerp and travels 580 miles to an airport in Berlin on a bearing of $N34°E$. The plane leaves the Berlin airport and travels to the Cairo airport 400 miles away on a bearing of $S74°E$. Find the distance between the airports in Antwerp and Cairo. Round to the nearest tenth of a mile.



$b^2 = 400^2 + 580^2 - 2(400)(580)\cos 108$

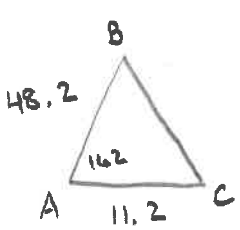
$b = \boxed{799.9 \text{ miles}}$

* dont know if A = 56°



5. Solve the triangle below. (Use Law of Sines and/or Law of Cosines). Round side lengths and angle measures to the nearest tenth.

$A = 162°,\ b = 11.2,\ c = 48.2$



$a^2 = 11.2^2 + 48.2^2 - 2(11.2)(48.2)\cos 162$

$a = 58.9$

* store full decimal

$\frac{11.2}{\sin B} = \frac{58.9}{\sin 162}$

$\sin B = \frac{11.2 \sin 162}{58.9}$

$B = 3.4°$

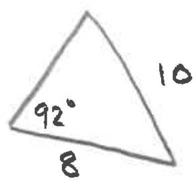
A = 162° a = $\underline{58.9}$

B = $\underline{3.4°}$ b = 11.2

C = $\underline{14.6°}$ c = 48.2

The Ambiguous Case (SSA) 2 sides given & one angle (not between sides)

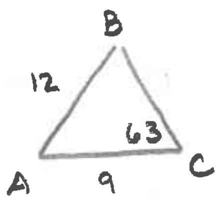
produces 0, 1, or 2 possible triangles



If the side opposite the given angle is bigger than the other side, only one Δ can be formed

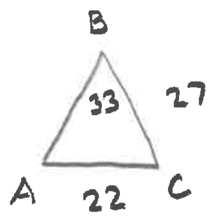
otherwise could be 0 or 2 Δs

6. Determine the number of triangles that can be formed with the given information.
   a. $b = 9,\ c = 12,\ C = 63^\circ$



$\boxed{\text{one } \Delta}$

   b. $a = 27,\ b = 22,\ B = 33^\circ$



0 or 2 Δ's

$$\frac{22}{\sin 33} = \frac{27}{\sin A}$$

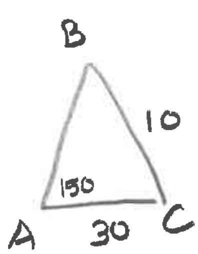
$$\sin A = \frac{27 \sin 33}{22}$$

$\sin A = 0.6684$

okay in domain

$\boxed{2\ \Delta\text{'s}}$

   c. $a = 10,\ b = 30,\ A = 150^\circ$



0 or 2 Δ's

$$\frac{10}{\sin 150} = \frac{30}{\sin B}$$

$$\sin B = \frac{30 \sin 150}{10}$$

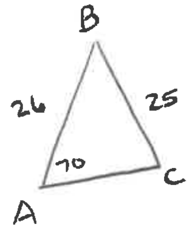
$\sin B = 1.5$

Not possible

$\boxed{0\ \Delta\text{'s}}$

7. Solve the triangle. If more than one solution is possible, find both solutions.

a. $a = 25,\ c = 26,\ A = 70°$



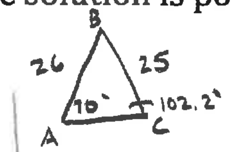
0 or 2 △'s

$\frac{25}{\sin 70} = \frac{26}{\sin C}$

$\sin C = \frac{26 \sin 70}{25}$

$\sin C = 0.977$

$C = 77.8°$

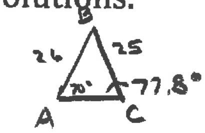


$\frac{25}{\sin 70} = \frac{b}{\sin 7.8}$

$B = 7.8°$

$b = 3.6$

$C = 102.2°$



$B = 32.2°$

$C = 77.8°$

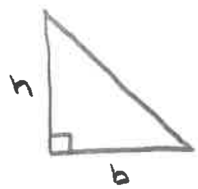
$b = 14.2$

S | A

or $C = 180 - 77.8°$

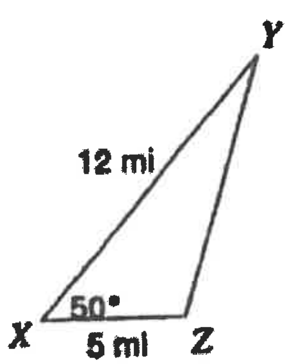
$C = 102.2°$

Area of oblique triangles

Right △	SAS	Heron's Formula
	$A = \frac{1}{2} bc \sin A$	$A = \sqrt{s(s-a)(s-b)(s-c)}$
	$= \frac{1}{2} ac \sin B$	where
	$= \frac{1}{2} ab \sin C$	$s = \frac{1}{2}(a+b+c)$
$A = \frac{1}{2} bh$		SSS

8. Find the area of each triangle below. Round your answer to the nearest hundredth.

a.



$A = \frac{1}{2}(5)(12)\sin 50$

$\approx 23\ mi^2$

b. $a = 145,\ b = 132,\ c = 84$

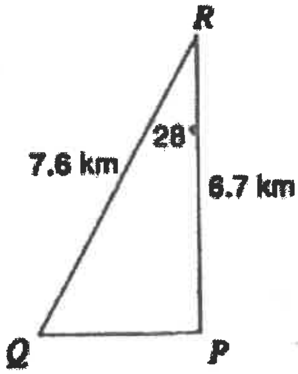
$s = \frac{1}{2}(145)(132)(84)$

$= 180.5$

$A = \sqrt{180.5(180.5 - 145)(180.5 - 132) \cdot (180.5 - 84)}$

$\approx 5476\ u^2$

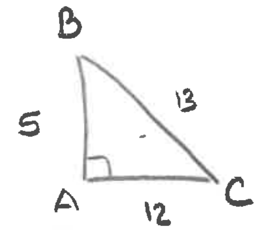
c.



$$A = \frac{1}{2}(6.7)(7.6)\sin(28)$$

$$\approx 12 \text{ km}^2$$

d. $a = 90^\circ,\ a = 13,\ b = 12$



$$A = \frac{1}{2}(5)(12)$$

$$= 30 \text{ u}^2$$