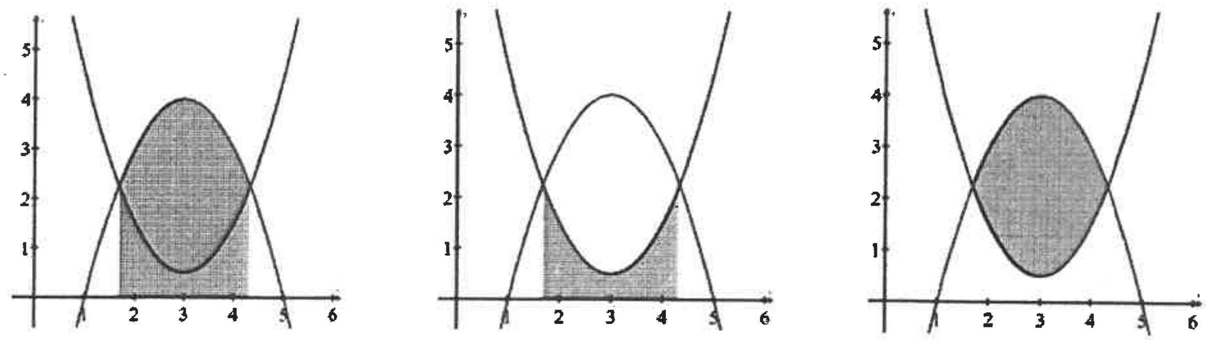
2018 - 19

BC Calculus

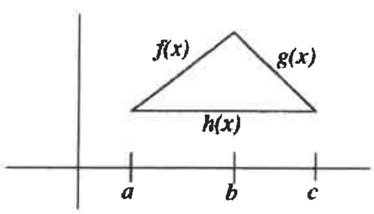
8.2 Areas in the Plane

We can extend the idea of definite integrals finding the area of a region *under* a curve to the area of a region *between* two curves. If two functions are both continuous on an interval [a, b], then the region between the curves can be found by **subtracting the area of the upper region and the area of the lower region.**



Area of upper function (-) Area of lower function = Area between the functions

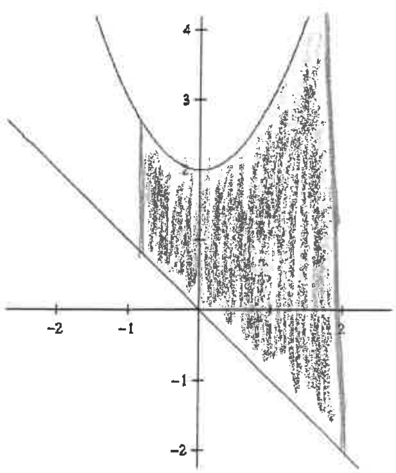
f(x) g(x) $\int_a^b (f(x)-g(x))dx$



Area from *a* to *c*

$$A = \int_a^b [f(x)-h(x)]\,dx + \int_b^c [g(x)-h(x)]\,dx$$

1. Find the area of the region bounded by $y = x^2 + 2$, $y = -x$, $x = -1$, $x = 2$.



$\int_{-1}^{2} \left[(x^2+2) - (-x)\right] dx$

$\int_{-1}^{2} (x^2 + x + 2)\,dx$

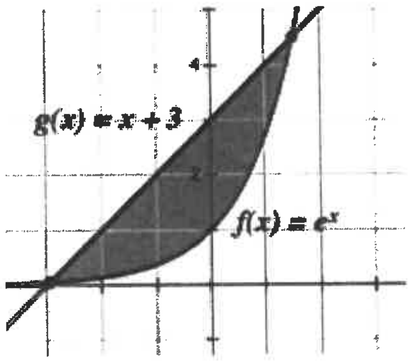
$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \Big|_{-1}^{2}$

$\frac{1}{3}(2)^3 + \frac{1}{2}(2)^2 + 2(2) - \left(\frac{1}{3}(-1)^3 + \frac{1}{2}(-1)^2 + 2(-1)\right)$

$\frac{8}{3} + 2 + 4 + \frac{1}{3} - \frac{1}{2} + 2$

$\boxed{10.5}$

2. Find the area of the region bounded by $f(x) = e^x$, $g(x) = x + 3$



$e^x = x + 3$

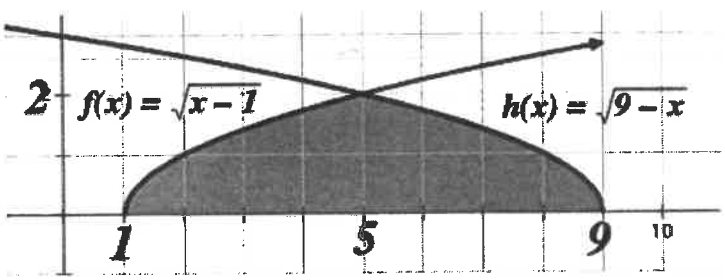
$0 = x + 3 - e^x$

$x_1 = -2.9475309$ and $x_2 = 1.505241495$

$$\int_{x_1}^{x_2} (x + 3 - e^x)\,dx$$

$\boxed{= 5.694}$

3. Find the area of the shaded region bounded by the x-axis and $f(x)$ and $h(x)$



$\sqrt{x-1} = \sqrt{9-x}$

$x - 1 = 9 - x$

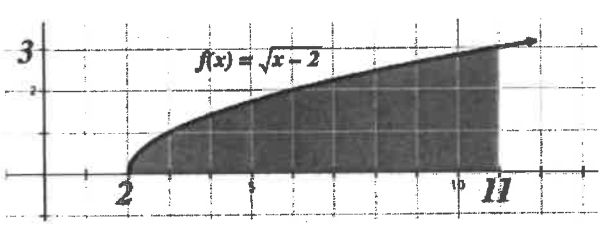
$2x = 10$

$x = 5$

$$\int_1^5 \sqrt{x-1}\,dx + \int_5^9 \sqrt{9-x}\,dx$$

$$\frac{2}{3}(x-1)^{3/2}\Big|_1^5 - \frac{2}{3}(9-x)^{3/2}\Big|_5^9 = \boxed{\frac{32}{3}}$$

4. Find the area of the shaded region from $[2, 11]$



$$\int_2^{11} \sqrt{x-2}\,dx$$

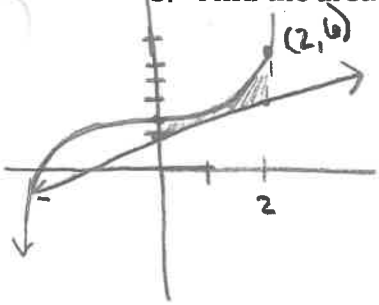
$$\frac{2}{3}(x-2)^{3/2}\Big|_2^{11}$$

$$\frac{2}{3}(11-2)^{3/2} - \frac{2}{3}(2-2)^{3/2}$$

$$\frac{2}{3}\sqrt{9}^{\,3} - 0$$

$\boxed{18}$

5. Find the area of the region bounded by $y = \frac{1}{2}x^3 + 2$ and $y = x + 1$ from $[0, 2]$



$$\int_0^2 \left[\frac{1}{2}x^3 + 2 - (x+1)\right] dx$$

$$= \int_0^2 \left(\frac{1}{2}x^3 - x + 1\right) dx$$

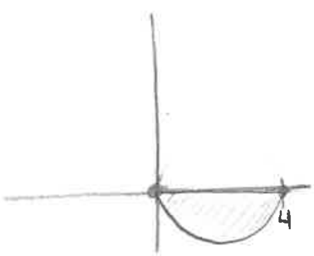
$$= \frac{1}{8}x^4 - \frac{1}{2}x^2 + x \Big|_0^2$$

$$= \left(\frac{1}{8}(2^4) - \frac{1}{2}(2)^2 + 2\right) - (0)$$

$$= \frac{1}{8}(16) - 2 + 2 = \boxed{2}$$

6. Find the area of the region bounded by $f(x) = x^2 - 4x$ and $g(x) = 0$

$f(x) = x(x-4)$



$$\int_0^4 (0 - (x^2 - 4x))\, dx$$

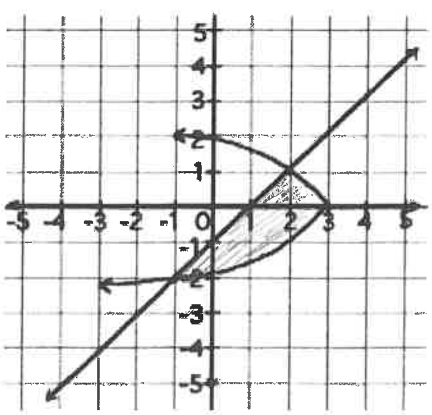
$$= -\left(\frac{1}{3}x^3 - 2x^2 \Big|_0^4\right)$$

$$= -\left[\frac{1}{3}(4)^3 - 2(4)^2\right] - (0)$$

$$= -\left(\frac{64}{3} - 32\right) = -\left(\frac{64}{3} - \frac{96}{3}\right) = \boxed{\frac{32}{3}}$$

7. Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $x = y + 1$

★ right − left ★



$$\int_{-2}^{1} \left[(3 - y^2) - (y + 1)\right] dy$$

$$= \int_{-2}^{1} (-y^2 - y + 2)\, dy$$

$$= -\frac{1}{3}y^3 - \frac{1}{2}y^2 + 2y \Big|_{-2}^{1}$$

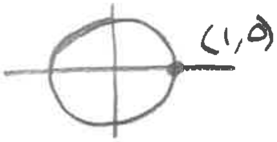
$3 - y^2 = y + 1$

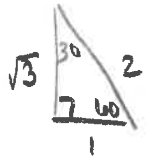
$0 = y^2 + y - 2$

$y = -2, 1$

$$= -\frac{1}{3} - \frac{1}{2} + 2 - \left(8/3 - 2 - 4\right)$$

$$= -9/3 - 1/2 + 8 = \boxed{4.5}$$





$\sin \pi/3 = \frac{\sqrt{3}}{2}$

$\tan \pi/3 = \sqrt{3}$

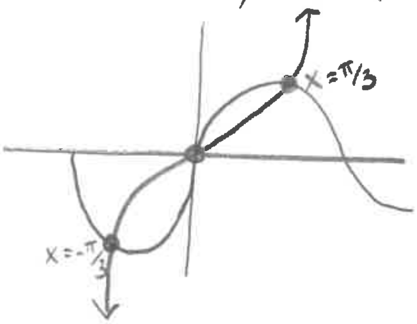
S | A
T | C

BC Calculus
8.2 Areas in the Plane

8. Find the area of the region bounded by $f(x) = 2\sin x$ and $g(x) = \tan x$

$2\sin x = \tan x$

$x = \pi/3,\ -\pi/3,\ 0$



$$\int_{-\pi/3}^{0} (\tan x - 2\sin x)\,dx + \int_{0}^{\pi/3} (2\sin x - \tan x)\,dx$$

$$\boxed{= 0.6137}$$

* enclosed area
so below x-axis not neg

9. Find the area of the region bounded by $f(x) = x^3 - 3x^2 + 3x$ and $g(x) = x^2$

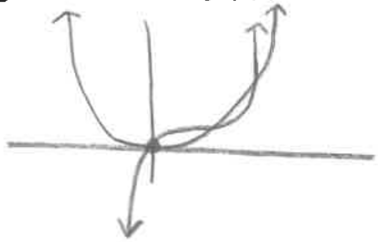
$x^2 = x^3 - 3x^2 + 3x$

$0 = x^3 - 4x^2 + 3x$

$0 = x(x^2 - 4x + 3)$

$0 = x(x-3)(x-1)$

$x = 0, 1, 3$

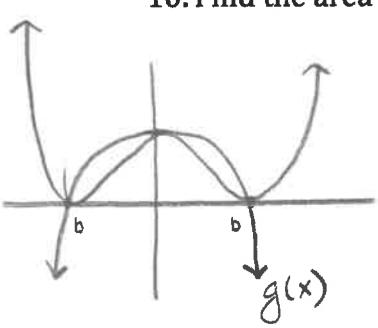


$$\int_{0}^{1} (x^3 - 3x^2 + 2x - (x^2))\,dx + \int_{1}^{3} (x^2 - (x^3 - 3x^2 + 3x))\,dx$$

$$= \boxed{\frac{37}{12}}$$

10. Find the area of the region bounded by $f(x) = x^4 - 2x^2 + 1$ and $g(x) = 1 - x^2$

$= (x^2 - 1)(x^2 - 1)$



$$2\int_{-1}^{0} \left[(1 - x^2) - (x^4 - 2x^2 + 1)\right] dx$$

$$= \boxed{\frac{4}{15}}$$