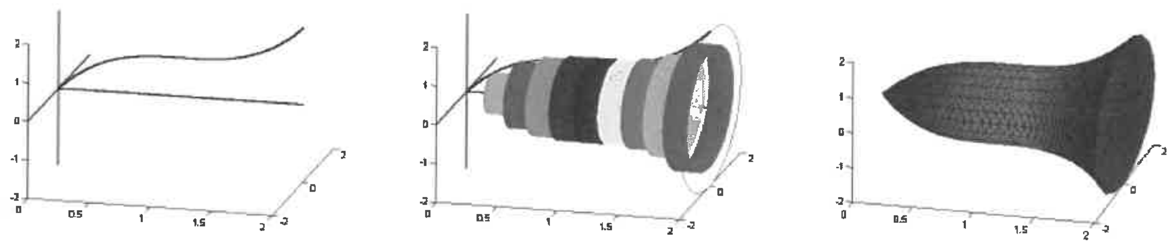
Volume by Disk Method

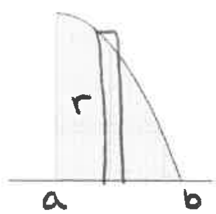
Create a 3-dimensional region is by rotating a function around a line. The rotation creates circular cross-sections that combine to create the volume. The resulting solid is called the **solid of revolution,** and the line that it revolved around is called the **axis of revolution.**
The area of each circle is $A = \pi r^2$, where r is distance from the function to the axis of revolution.

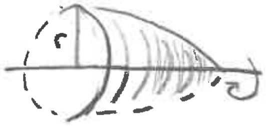


To find the volume of a solid of revolution with the disk method, use one of the following formulas:

<u>Horizontal Axis of Revolution</u>

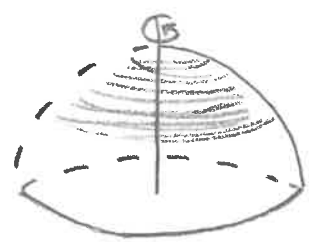
$$V = \int_a^b \pi r^2 \, dx$$





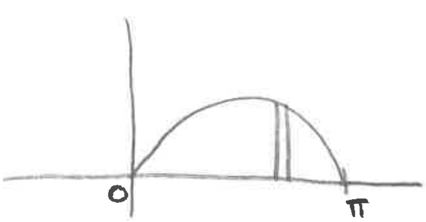
<u>Vertical Axis of Revolution</u>

$$V = \int_c^d \pi r^2 \, dy$$



2. **Using the Disk Method x-axis**

Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = \sqrt{\sin x}$ and the x-axis from $[0, \pi]$ about the x-axis.



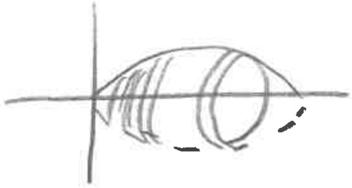
$$A = \pi(\sqrt{\sin x})^2$$

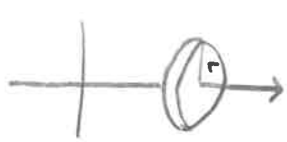
$$V = \int_0^\pi \pi(\sqrt{\sin x})^2 \, dx$$

$$= \pi \int_0^\pi \sin x \, dx$$

$$= -\pi \cos x \Big|_0^\pi$$

$$\boxed{= 2\pi}$$



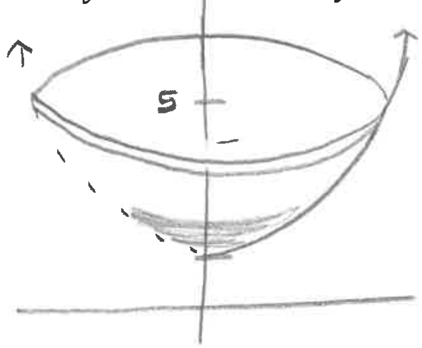


$r = f(x)$
$= \sqrt{\sin x}$

$A = \pi r^2$

3. **Using the Disk Method y-axis**

Find the volume of the solid formed by revolving the region bounded by the graph of $y = x^2 + 1$ and the y-axis for $1 \le y \le 5$ about the y-axis.



$$V = \int_1^5 \pi \left(\sqrt{y-1}\right)^2 dy$$

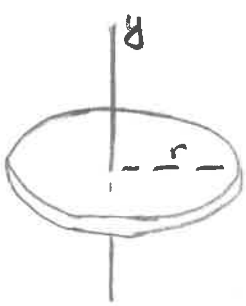
$$= \pi \int_1^5 (y-1)\, dy$$

$$= \pi \left(y^2/2 - y \Big|_1^5 \right)$$

$$= \pi \left[\left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right) \right]$$

$$= \pi (12 - 4)$$

$$\boxed{= 8\pi}$$



$r = x^2 + 1$

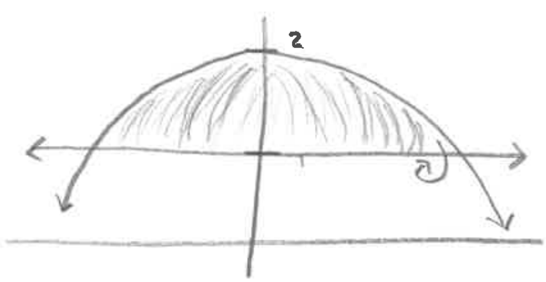
* need in terms of y

$y = x^2 + 1$

$\sqrt{y-1} = x = r$

4. **Revolving About a Line That is Not a Coordinate Axis**

Find the volume of the solid formed by revolving the region bounded by $f(x) = 2 - x^2$ and $g(x) = 1$ about the line $y = 1$.



intersection points

$2 - x^2 = 1$

$x^2 = 1$

$x = \pm 1$

$A(x) = \pi (1 - x^2)^2$

$$\int_{-1}^{1} \pi (1-x^2)^2\, dx$$

$$\boxed{= \frac{16}{15}\pi}$$

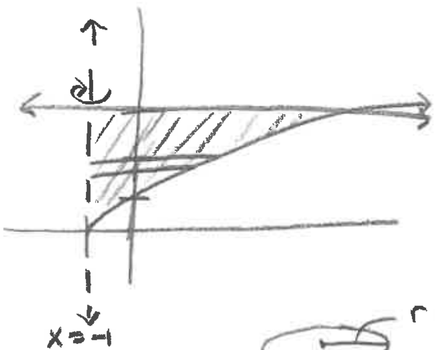
$r = (2 - x^2) - (1)$

$r = 1 - x^2$

Find the volume of the solid formed by revolving the region bounded by $y = \sqrt{x+1}$, $y = 3$, and $x = -1$ about the line $x = -1$.

★need in terms of y!

$y^2 - 1 = x$



$r = (y^2 - 1) - (-1)$

$= y^2$

$$V = \int_0^3 \pi (y^2)^2 \, dy$$

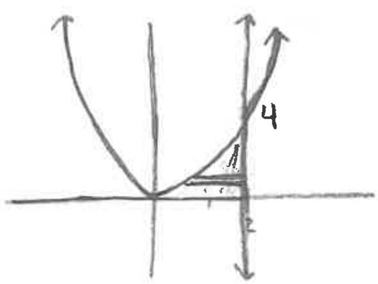
$$= \frac{\pi}{5} y^5 \Big|_0^3$$

$$= \frac{\pi}{5}(3)^5 - \frac{\pi}{5}(0)^5$$

$$\boxed{= \frac{243}{5}\pi}$$

Find the volume of the solid formed by revolving the region bounded by $y = x^2$, the x-axis, and $x = 2$ about the line $x = 2$.

$x = \sqrt{y}$ ★ only want positive



$$\int_0^4 \pi (2 - \sqrt{y})^2 \, dy$$

$$\boxed{= \frac{8}{3}\pi}$$

intersection

$\sqrt{y} = 2$

$y = 4$

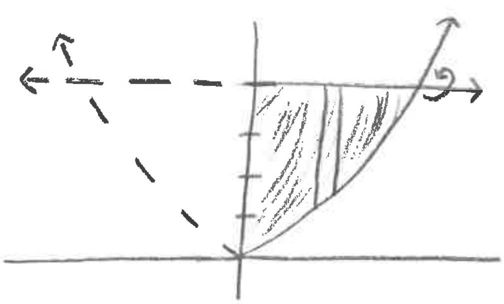
$r = 2 - \sqrt{y}$

Find the volume of the solid formed by revolving the region bounded by $y = x^2$, $y = 4$, and the y-axis, about the line $y = 4$.

intersection

$4 = x^2$

$\pm 2 = x$



$$V = \int_0^2 \pi (4 - x^2)^2 \, dx$$

$$\boxed{= \frac{256\pi}{15}}$$

$r = 4 - x^2$