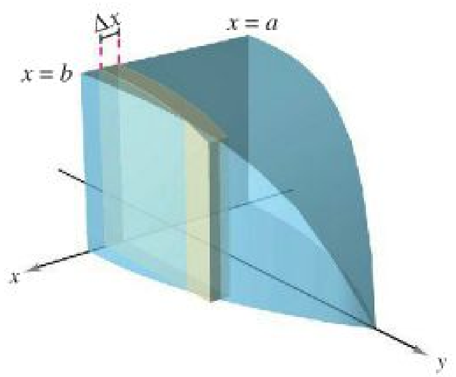
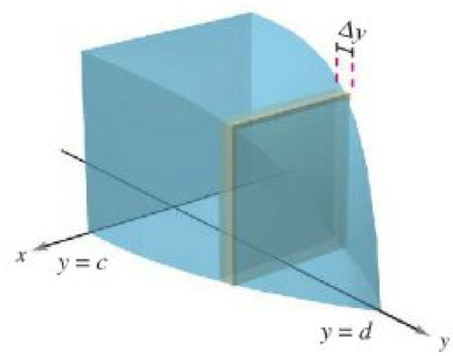
Volume by Cross Sections

Two-dimensional cross sections can be stacked on top of (or next to) each other to create a 3D figure. We use the area between two curves to represent the base of the cross sectional shape.





Volume of a Solid:

Cross Sections taken **PERPENDICULAR** to the **X-AXIS**
The volume of a solid of known integrable cross section area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b.

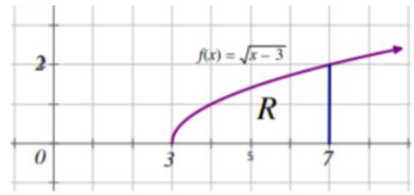
$$V = \int_a^b A(x)dx.$$

Cross Sections taken **PERPENDICULAR** to the **Y-AXIS**
The volume of a solid of known integrable cross section area $A(y)$ from $y = c$ to $y = d$ is the integral of A from c to d

$$V = \int_C^b A(y)dy.$$

Where $A(x)$ and $A(y)$ are known geometric formulas for the area of a shape.
Example: square cross-sections $\rightarrow A = s^2$, semi-circle cross sections $\rightarrow A = \frac{1}{2}\pi r^2$, equilateral triangle cross-sections $\rightarrow A = \frac{\sqrt{3}}{4}s^2$

1. Let R be the region in the first quadrant under the graph of $y = \sqrt{x-3}$ for $3 \le x \le 7$. Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis (vertical cross sections) are:



   a. Squares

   b. Equilateral Triangles

   c. Semicircles

   d. Rectangle with height of 5

e. Isosceles Right Triangle where the leg is the base.

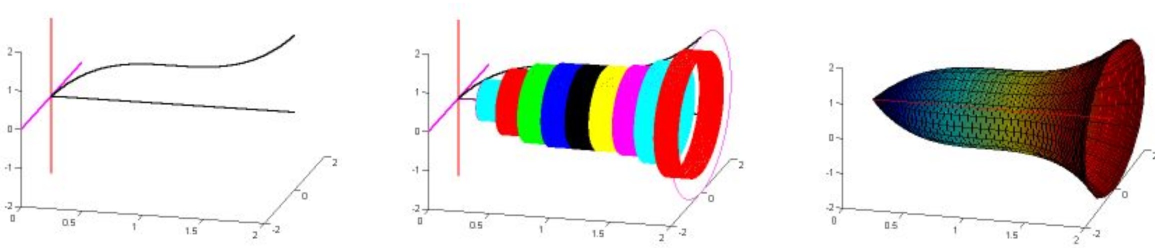
2. Let R be the region in the first quadrant under the graph of $y = \sqrt{x-3}$ for $3 \leq x \leq 7$. Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the y-axis (horizontal cross sections) are:
   a. Squares

   b. Isosceles Right Triangles where the hypotenuse is the base

Volume by Disk Method

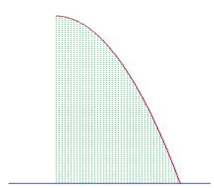
Create a 3-dimensional region is by rotating a function around a line. The rotation creates circular cross-sections that combine to create the volume. The resulting solid is called the **solid of revolution**, and the line that it revolved around is called the **axis of revolution**.

The area of each circle is $A = \pi r^2$, where r is distance from the function to the axis of revolution.

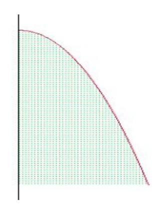


To find the volume of a solid of revolution with the disk method, use one of the following formulas:

Horizontal Axis of Revolution



Vertical Axis of Revolution



2. **Using the Disk Method x-axis**

Find the volume of the region enclosed by $f(x) = \sqrt{x}$ and the x-axis from $[0, 9]$ rotated about the x-axis.

3. **Using the Disk Method y-axis**

Find the volume of the solid generated when the area bounded by lines $y = 6x - 3$, $x = 0$, and $y = 6$ is revolved around the y –axis.

4. **Revolving About a Line That is Not a Coordinate Axis**

Find the volume of the solid formed by revolving the region bounded by $f(x) = 2 - x^2$ and g(x) = -1 about the line y = -1.

Find the volume of the solid formed by revolving the region bounded by y = $\sqrt{x+4}$, y = 2, and x = 2 about the line x = 2.

Volume by Washer Method

The disk method can be extended to cover solids of revolution with holes by replacing the representative disk with a representative **washer**. The washer is formed by revolving a rectangle about an axis.

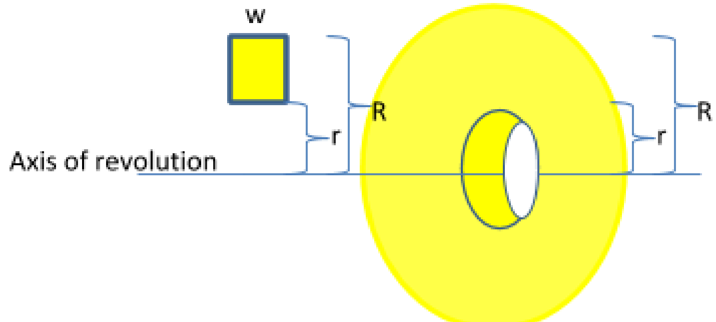
Washer Method:

If r is the inner radius and R is the outer radius, then

$$V = \pi\int_a^b [R(x)^2 - r(x)^2]dx$$

or,

$$V = \pi\int_a^b R(x)^2dx - \pi\int_a^b r(x)^2dx$$



8. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = 4x - x^2$ and $f(x) = x^2$ from $[0, 2]$ about the…

a) x-axis.

b) the line $y = 5$

9. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = 2x + 2$, $y = x^2 + 2$ about the x-axis.

10. Find the volume of the solid generated by revolving the region bounded by $x = -y^2 + 2$, $x = y$ about the line $x = -3$.