

### Volume by Washer Method

The disk method can be extended to cover solids of revolution with holes by replacing the representative disk with a representative **washer**. The washer is formed by revolving a rectangle about an axis.

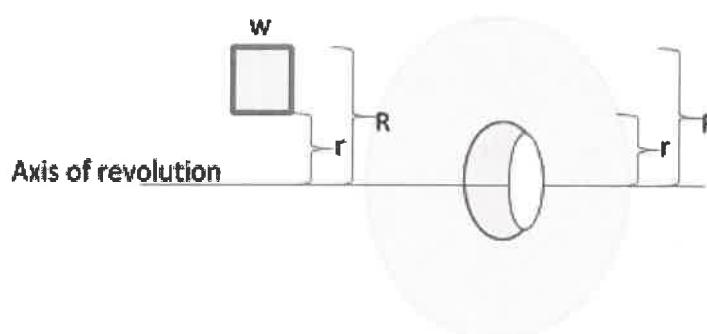
#### Washer Method:

If  $r$  is the inner radius and  $R$  is the outer radius, then

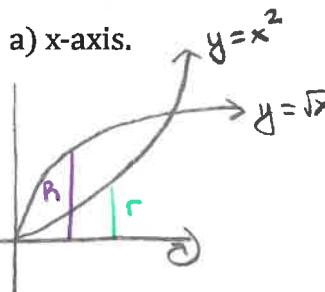
$$V = \pi \int_a^b [R(x)^2 - r(x)^2] dx$$

or,

$$V = \pi \int_a^b R(x)^2 dx - \pi \int_a^b r(x)^2 dx$$



8. Find the volume of the solid formed by revolving the region bounded by the graphs  $y = x^2$  of  $y = \sqrt{x}$  and  $y = x$  about the...



$$R(x) = \sqrt{x}$$

$$A_R(x) = \pi(\sqrt{x})^2$$

$$r(x) = x^2$$

$$A_r(x) = \pi(x^2)^2$$

intersection

$$\sqrt{x} = x^2$$

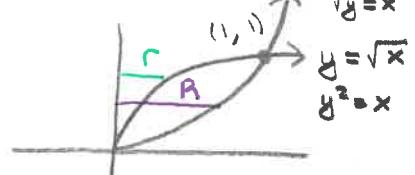
$$x = x^4$$

$$0 = x^4 - x$$

$$0 = x(x^3 - 1)$$

$$x = 0, 1$$

b) y-axis



$$R(y) = \sqrt{y}$$

$$r(y) = y^2$$

$$A_R(y) = \pi(\sqrt{y})^2$$

$$A_r(y) = \pi(y^2)^2$$

$$V = \int_0^1 \pi(\sqrt{y})^2 dy - \int_0^1 \pi(y^2)^2 dy$$

$$= \frac{3}{10}\pi$$

$$V = \int_0^1 \pi(\sqrt{x})^2 dx - \int_0^1 \pi(x^2)^2 dx$$

$$= \int_0^1 \pi x dx - \int_0^1 \pi x^4 dx$$

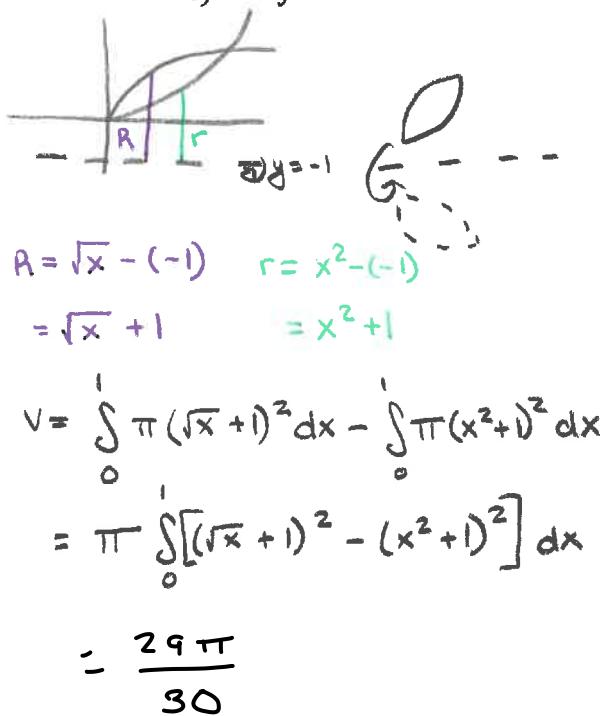
$$= \pi \left[ \frac{x^2}{2} \Big|_0^1 - \frac{x^5}{5} \Big|_0^1 \right]$$

$$= \pi [(1/2) - (1/5)]$$

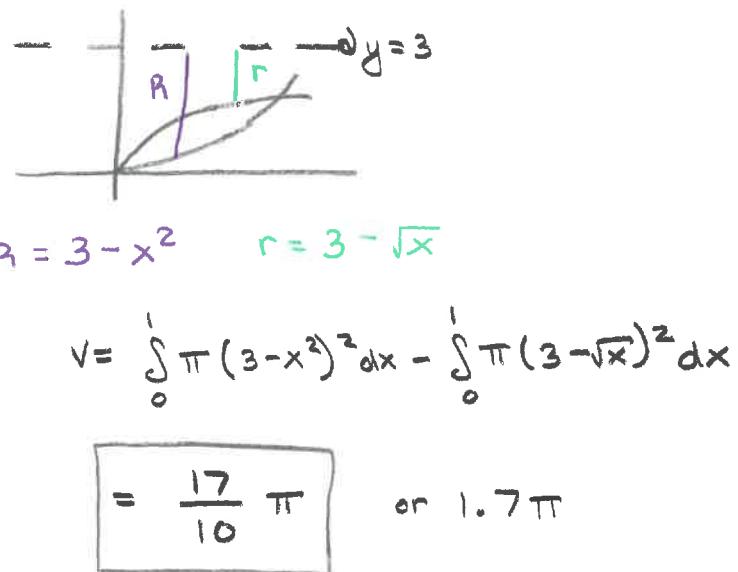
$$= \frac{3}{10}\pi$$

AB Calculus  
8.3 Volumes (Washers)

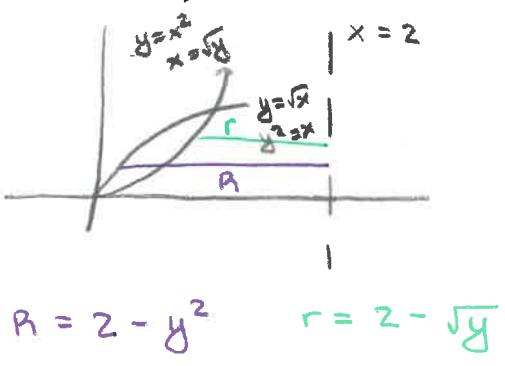
c) line  $y = -1$



d) line  $y = 3$



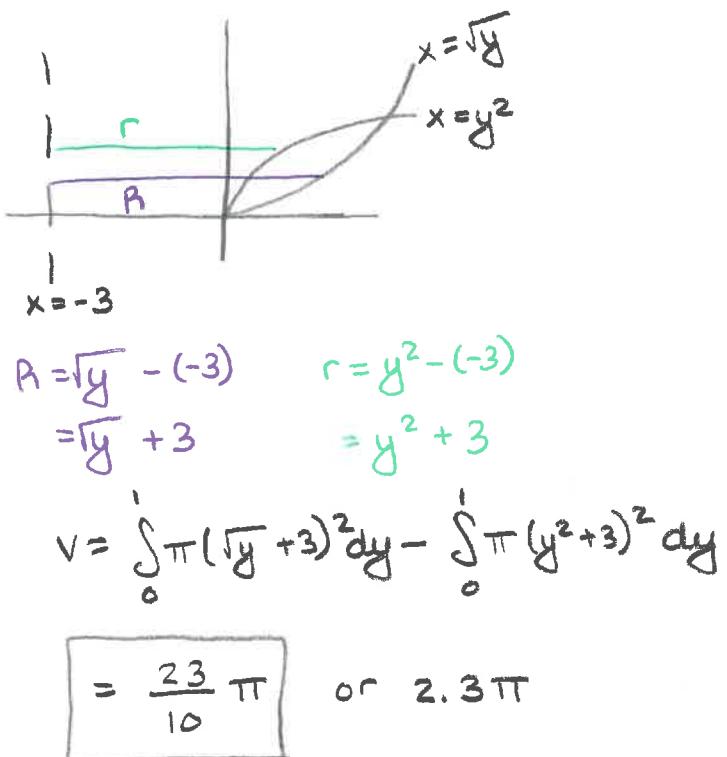
e) the line  $x = 2$



$$V = \int_0^4 \pi(2-y^2)^2 dy - \int_0^4 \pi(2-\sqrt{y})^2 dy$$

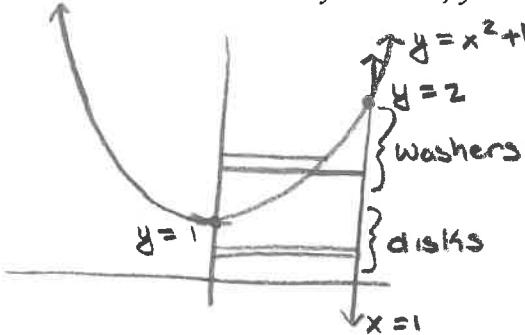
$$= 1.033\pi$$

f) the line  $x = -3$



AB Calculus  
8.3 Volumes (Washers)

9. Find the volume of the solid formed by revolving the region bounded by the graphs of  $y = x^2 + 1$ ,  $y = 0$ ,  $x = 0$ , and  $x = 1$  about the y-axis.



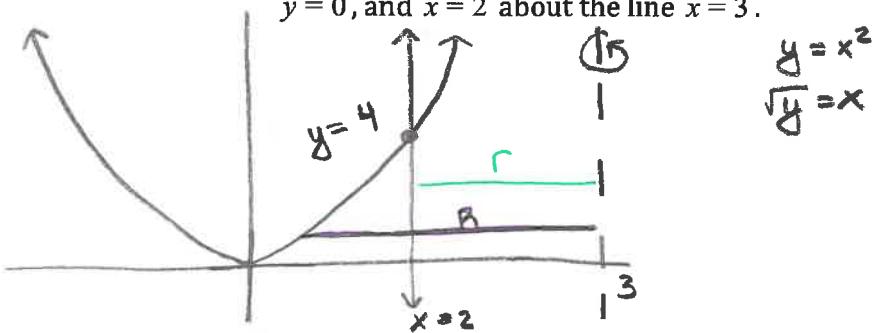
$$y = x^2 + 1$$

$$\sqrt{y-1} = x$$

$$V = \underbrace{\int_0^1 \pi(1)^2 dy}_{\text{disks}} + \left[ \underbrace{\int_1^2 \pi(1)^2 dy}_{\text{washers}} - \int_1^2 \pi(\sqrt{y-1})^2 dy \right]$$

$$V = 1.5 \pi$$

10. Find the volume of the solid generated by revolving the region bounded by  $y = x^2$ ,  $y = 0$ , and  $x = 2$  about the line  $x = 3$ .



$$V = \int_0^4 \pi(3 - \sqrt{y})^2 dy - \int_0^4 \pi(3 - 2)^2 dy$$

$$= \int_0^4 \pi(3 - \sqrt{y})^2 dy - \int_0^4 \pi dy$$

$$= 8\pi$$

