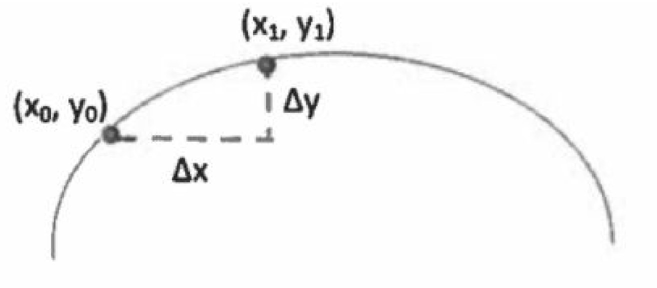
Arc Length:
The length of a curve is found by summing small segments using distance formula



$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2\left(\frac{\Delta x}{\Delta x}\right)^2}$$

$$d = \sqrt{(\Delta x)^2 + \left(\frac{\Delta y}{\Delta x}\right)^2 (\Delta x)^2}$$

$$d = \sqrt{(\Delta x)^2\left(1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right)}$$

$$d = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\,(\Delta x)$$

$$d = \sqrt{1 + f'(x)^2}(\Delta x)$$

$$length = \int_a^b \sqrt{1 + f'(x)^2}dx$$

Smoothness:

A function with a continuous first derivative is **smooth** and its graph is a **smooth curve.**

DEFINITION **Arc Length: Length of a Smooth Curve**

If a smooth curve begins at (a, c) and ends at (b, d), $a < b$, $c < d$, then the **length (arc length) of the curve** is

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad \text{if } y \text{ is a smooth function of } x \text{ on } [a, b];$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \qquad \text{if } x \text{ is a smooth function of } y \text{ on } [c, d].$$

1. Find the arc length of $f(x) = x^2$ on the interval of $[-1, 1]$.

2. Find the arc length of $f(x) = \frac{3}{2}x^{\frac{2}{3}}$ on the interval of $[1,\ 8]$.

Getting around a corner:

3. Find the length of the curve $y = x^2 - 4|x| - x$ from $x = -4$ to $x = 4$.

4. Find the length of the curve $x = \int_0^y \sqrt{\sec^2 t - 1}\, dt$, $-\frac{\pi}{3} \le y \le \frac{\pi}{4}$

5. Find the length of the curve $y = x^{\frac{1}{3}}$ on the interval of $[-8,\ 8]$