BC Calculus
9.1 Sequences

Sequence: is a list of numbers written in an explicit order.

Primary focus is on infinite sequences and whether or not they converge or diverge. If a sequences converges its terms approach limiting values.

Limit of a Sequence

We write $\lim_{n\to\infty} a_n = L$ and say that the sequence converges to L. Sequences that do not have limits diverge

Properties of Limits

If L and M are real numbers and $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = M$, then:

1. Sum Rule:
   $\lim_{n\to\infty}(a_n + b_n) = L + M$
2. Product Rule:
   $\lim_{n\to\infty}(a_n b_n) = L \bullet M$
3. Quotient Rule:
   $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{L}{M}$
4. Difference Rule:
   $\lim_{n\to\infty}(a_n - b_n) = L - M$
5. Constant Multiple Rule:
   $\lim_{n\to\infty}(c \bullet a_n) = c \bullet L$

1. Determine whether the sequence converges or diverges. If it converges, find its limit.
   a. $a_n = \frac{2_n - 1}{n}$
   b. $a_n = \frac{n}{n^2+1}$

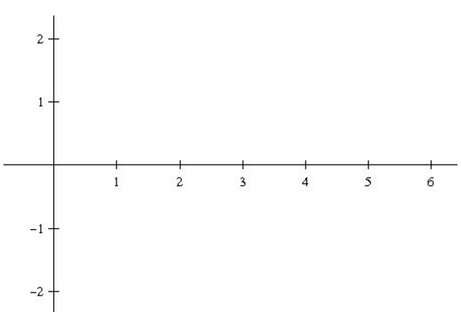
c. $a_n = (-1)^n \frac{n+1}{n^2+2}$

d. $a_n = (0.9)^n$

e. $a_n = \cos(n\frac{\pi}{2})$

Squeeze Theorem for Sequences

If $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$ and if there is an integer N for which $a_n \le b_n \le c_n$ for all $n > N$, then $\lim_{n\to\infty} b_n = L$



2. Use the Squeeze Theorem to show that the sequence with given nth term converges and find its limit.

a. $a_n = \frac{1}{2^n}$

b. $a_n = \frac{\sin^2 n}{2^n}$

Absolute Value Theorem
Consider the sequence $\{ a_n \}$. If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$