

L'Hospital's Rule

Suppose that $f(a) = g(a) = 0$, that f and g are differentiable on an open interval I containing a and that $g'(a) \neq 0$ on I if $x \neq a$. Then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(a)}{g'(a)}$$

if the latter limit exists.

* $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(a)}{g'(a)}$ can be used for the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$.

1. Evaluate the following:

a. $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$ check: $\frac{0}{0}$
 $\lim_{x \rightarrow 1} \frac{3x^2}{1}$
 $= \frac{3}{1}$
 $= 3$

b. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x}{\sin x - 1}$ $\frac{0}{0}$
 $\lim_{x \rightarrow \frac{\pi}{2}} \frac{2\cos x(-\sin x)}{\cos x}$
 $= \lim_{x \rightarrow \frac{\pi}{2}} (-2\sin x)$
 $= -2$

c. $\lim_{x \rightarrow 0^+} x \ln x = \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x} = \frac{-\infty}{\infty}$
 $\lim_{x \rightarrow 0^+} \frac{1/x}{-1/x^2}$
 $= \lim_{x \rightarrow 0^+} \frac{1}{x} (-x^2)$
 $= \lim_{x \rightarrow 0^+} (-x) = 0$

d. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin \theta}{1 + \cos 2\theta}$ $\frac{0}{0}$
 $\lim_{x \rightarrow \frac{\pi}{2}} \frac{-\cos \theta}{-2\sin 2\theta}$
 $= \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\sin \theta}{4\cos 2\theta}$
 $= \frac{-1}{4(-1)} = \frac{1}{4}$

When we reach a point where one of the derivatives approaches 0 and the other does not, then:

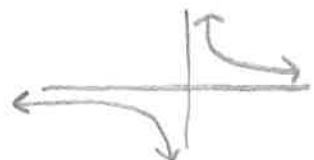
- a. The numerator approaches 0 \rightarrow the limit is 0
- b. The denominator approaches 0 \rightarrow the limit is $\pm\infty$

2. Evaluate the following $\lim_{x \rightarrow 0} \frac{\sin x}{x^2}$ $\frac{0}{0}$

$$\lim_{x \rightarrow 0} \frac{\cos x}{2x} = \frac{1}{0}$$

L'Hospital's doesn't apply

must use other methods



$$\lim_{x \rightarrow 0} \frac{\sin x}{x^2} = \text{DNE}$$

9.2 L'Hospital's Rule

AB Calculus

3. Evaluate the following:

a. $\lim_{x \rightarrow 0^-} \frac{\tan x}{x} = \frac{0}{0}$

$$\lim_{x \rightarrow 0^-} \frac{\sec^2 x}{1}$$

$$= \sec^2(0)$$

$$= 1$$

b. $\lim_{x \rightarrow 0^+} \frac{\tan x}{x} = \frac{0}{0}$

$$\lim_{x \rightarrow 0^+} \frac{\sec^2 x}{1}$$

$$= \sec^2(0)$$

$$= 1$$

Evaluate the following:

4. $\lim_{y \rightarrow 0^+} \frac{\ln(y^2+2y)}{\ln y} = \frac{-\infty}{-\infty}$

$$\lim_{y \rightarrow 0^+} \frac{\frac{1}{y^2+2y} (2y+2)}{\frac{1}{y}}$$

$$= \lim_{y \rightarrow 0^+} \frac{2y+2}{y^2+2y} \cdot \frac{1}{y}$$

$$= \lim_{y \rightarrow 0^+} \frac{2y+2}{y^2+2y}$$

$$= \frac{2}{2}$$

$$= 1$$

5. $\lim_{x \rightarrow 1} \frac{\int_1^x \frac{dt}{t}}{x^3-1} = \frac{0}{0}$

$$\lim_{x \rightarrow 1} \frac{\frac{1}{3}x^2}{3x^2}$$

$$= \frac{1}{3}$$

9.2 L'Hospital's Rule

AB Calculus

$$6. \lim_{x \rightarrow \infty} \frac{x^2}{e^x} \quad \frac{\infty}{\infty}$$

\Rightarrow

$$= \lim_{x \rightarrow \infty} \frac{2x}{e^x} \quad \frac{0}{\infty}$$

\Rightarrow

$$= \lim_{x \rightarrow \infty} \frac{2}{e^x} \quad 0$$

$$\frac{2}{\infty} \quad 0$$

= 0

$$8. \lim_{x \rightarrow -\infty} \frac{3x-1}{1-5x} \quad \frac{\infty}{\infty}$$

\Rightarrow

$$= \lim_{x \rightarrow -\infty} \frac{3}{-5} \quad -\frac{3}{5}$$

= -\frac{3}{5}

$$7. \lim_{x \rightarrow \infty} \frac{e^{2x}-1}{x} \quad \frac{\infty}{\infty}$$

\Rightarrow

$$= \lim_{x \rightarrow \infty} \frac{2e^{2x}}{1} \quad \infty$$

= \infty

$$9. \lim_{x \rightarrow \infty} \frac{3x^3+4x^2}{4x^3-7} \quad \frac{\infty}{\infty}$$

\Rightarrow

$$= \lim_{x \rightarrow \infty} \frac{9x^2+8x}{12x^2} \quad \frac{0}{\infty}$$

\Rightarrow

$$= \lim_{x \rightarrow \infty} \frac{18x+8}{24x} \quad \frac{0}{\infty}$$

\Rightarrow

$$= \lim_{x \rightarrow \infty} \frac{18}{24} \quad \frac{0}{0}$$

$= \frac{18}{24}$

= $\frac{3}{4}$

9.2 L'Hospital's Rule

AB Calculus

$$10. \lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x}$$