Indeterminate Forms:

$\frac{0}{0}$ $\frac{\infty}{\infty}$ $\infty - \infty$ $0(\infty)$ 0^0 1^∞ ∞^0

*Put the limit in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ if it is in a different indeterminate form before apply l'hospital's rule.

L'hospital's Rule:

$$\text{If } \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0} \text{ or } \frac{\infty}{\infty}, \text{ then } \lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

1. $\lim_{x\to 2}\frac{x^2+3x-10}{x-2}$ $\frac{0}{0}$

$\lim_{x\to 2}\frac{2x+3}{1} = 2(2)+3 = \boxed{7}$

2. $\lim_{x\to 0}\frac{\sin x}{x}$ $\frac{0}{0}$

$\lim_{x\to 0}\frac{\cos x}{1} = \cos 0 = \boxed{1}$

3. $\lim_{x\to \infty}\frac{e^x}{x^3}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{e^x}{3x^2}$

$\lim_{x\to\infty}\frac{e^x}{6x}$

$\lim_{x\to\infty}\frac{e^x}{6} = \frac{e^\infty}{6} = \boxed{\infty}$

4. $\lim_{x\to\infty} \frac{x^2}{e^x-1}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{2x}{e^x}$

$\lim_{x\to\infty} \frac{2}{e^x} = \frac{2}{\infty} = \boxed{0}$

5. $\lim_{x\to 0} \frac{3e^x-3x-3}{x^2}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{3e^x - 3}{2x}$

$\lim_{x\to\infty} \frac{3e^x}{2} = \frac{3e^\infty}{2} = \boxed{\infty}$

6. $\lim_{x\to 0^+} x \ln x$ $0\cdot(-\infty)$

* rewrite

$\lim_{x\to 0^+} \frac{\ln x}{1/x}$ $\frac{-\infty}{\infty}$

$\lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} -\frac{x^2}{x} = \lim_{x\to 0^+} -x = \boxed{0}$

7. $\lim_{x\to\infty} x^{1/x}$ ∞^0

* $x^{1/x} = e^{\ln x^{1/x}}$

$= e^{\frac{\ln x}{x}}$

$= \lim_{x\to\infty} e^{\frac{\ln x}{x}} = e^{\lim_{x\to\infty} \frac{\ln x}{x}}$ $\frac{\infty}{\infty}$

$= e^{\lim_{x\to\infty} \frac{1/x}{1}} = e^0 = \boxed{1}$

Key

$\lim_{x\to b} a^{f(x)} = a^{\lim_{x\to b} f(x)}$

$\lim_{x\to b} \log_a f(x) = \log_a \left(\lim_{x\to b} f(x)\right)$

8. $\lim_{x\to 3^+} \frac{18}{x^2-9} - \frac{x}{x-3}$

$= \lim_{x\to 3^+} \frac{18}{x^2-9} - \frac{x(x+3)}{x-3(x+3)}$

$= \lim_{x\to 3^+} \frac{18 - x^2 - 3x}{x^2-9}$

$= \lim_{x\to 3^+} \frac{-(x^2+3x-18)}{x^2-9}$ $\frac{0}{0}$

$\lim_{x\to 3^+} \frac{-2x-3}{2x}$

$= \frac{-6-3}{6}$

$= \frac{-3}{2}$