L'Hospital's Rule
Suppose that $f(a) = g(a) = 0$, that f and g are differentiable on an open interval I containing a and that $g'(a) \neq 0$ on I if $x \neq a$. Then
$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(a)}{g'(a)}$$
if the latter limit exists.

* $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(a)}{g'(a)}$ can be used for the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$

1. Evaluate the following:
   a. $\lim_{x\to 1} \frac{x^3-1}{x-1}$
   c. $\lim_{x\to 0^+} x \ln x$
   b. $\lim_{x\to \frac{\pi}{2}} \frac{\cos^2 x}{\sin x - 1}$
   d. $\lim_{x\to \frac{\pi}{2}} \frac{1-\sin\theta}{1+\cos 2\theta}$

When we reach a point where one of the derivatives approaches 0 and the other does not, then:

a. The numerator approaches 0 → the limit is 0
b. The denominator approaches 0 → the limit is $\pm\infty$

2. Evaluate the following $\lim_{x\to 0} \frac{\sin x}{x^2}$

3. Evaluate the following:

a. $\lim_{x \to 0^-} \frac{\tan x}{x}$

b. $\lim_{x \to 0^+} \frac{\tan x}{x}$

Evaluate the following:

4. $\lim_{y \to 0^+} \frac{\ln(y^2+2y)}{\ln y}$

5. $\lim_{x \to 1} \frac{\int_1^x \frac{dt}{t}}{x^3-1}$

6. $\lim\limits_{x \to \infty} \frac{x^2}{e^x}$

7. $\lim\limits_{x \to \infty} \frac{e^{2x}-1}{x}$

8. $\lim\limits_{x \to -\infty} \frac{3x-1}{1-5x}$

9. $\lim\limits_{x \to \infty} \frac{3x^3+4x^2}{4x^3-7}$

10. $\lim_{x \to \infty} \frac{(\ln x)^2}{x}$