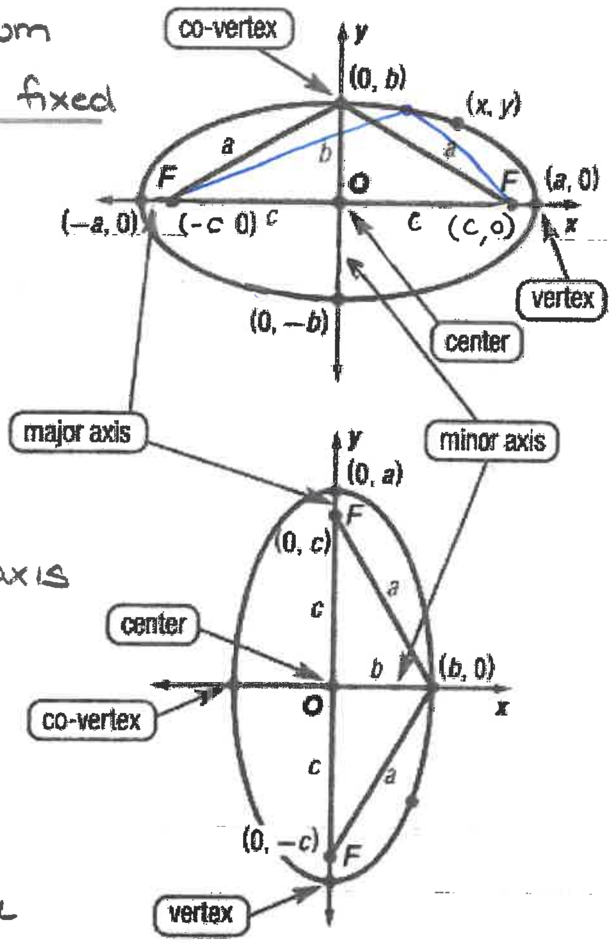
Ellipse: the set of all points in a plane such that the sum of the distances from two fixed points is constant

foci



Major and Minor Axis:

The 2 axis of symmetry ⊥ @ the center

*foci always on major axis

Constant Sum

the sum of the distances from the foci to any point on the ellipse

KeyConcept Equations of Ellipses Centered at the Origin		
Standard Form	stretch in x direction → $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ← stretch in y direction	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Orientation	horizontal	vertical
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Length of Major Axis	$2a$ units	$2a$ units
Length of Minor Axis	$2b$ units	$2b$ units

Important Relationships:

* The length of the major axis equals the constant sum

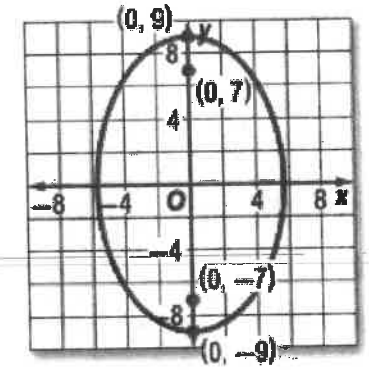
* $c^2 = a^2 - b^2$

focus^2 = $(\frac{1}{2}$ major axis$)^2$ − $(\frac{1}{2}$ minor axis$)^2$

↓ x-value

* distance from a focus to either co-vertex equals $\frac{1}{2}$ (major axis)

1. Write the equation for the ellipse:



center → (0,0)

so foci $c = 7$

$a = 9$

$b = ?$

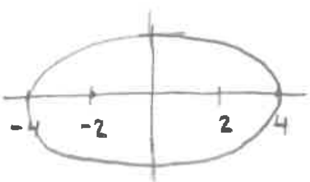
$c^2 = a^2 - b^2$

$7^2 = 9^2 - b^2$

$b = \sqrt{32}$

$$\boxed{\frac{x^2}{32} + \frac{y^2}{81} = 1}$$

2. Write an equation for an ellipse with vertices at (-4, 0) and (4, 0) and foci at (2, 0) and (-2, 0).



$c = 2$

$a = 4$

$2^2 = 4^2 - b^2$

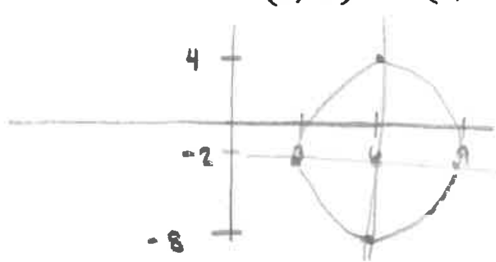
$4 = 16 - b^2$

$b = \sqrt{12}$

$$\boxed{\frac{x^2}{16} + \frac{y^2}{12} = 1}$$

KeyConcept Equations of Ellipses Centered at (h, k)		
Standard Form	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Orientation	horizontal	vertical
Foci	$(h \pm c, k)$	$(h, k \pm c)$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Co-vertices	$(h, k \pm b)$	$(h \pm b, k)$

3. Write an equation for the ellipse with vertices at (6, -8) and (6, 4) and co-vertices at (3, -2) and (9, -2).



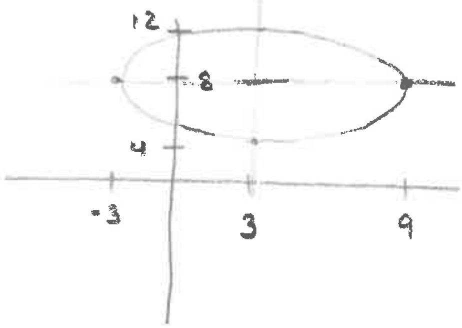
$b = \frac{1}{2}(8+4) = 6$

$a = \frac{1}{2}(9-3) = 3$

center $(6, -2)$

$$\boxed{\frac{(x-6)^2}{9} + \frac{(y+2)^2}{36} = 1}$$

4. Write an equation for the ellipse with vertices at (-3, 8) and (9, 8) and co-vertices at (3, 12) and (3, 4).



center (3, 8)

$a = \frac{1}{2}(9-(-3))$

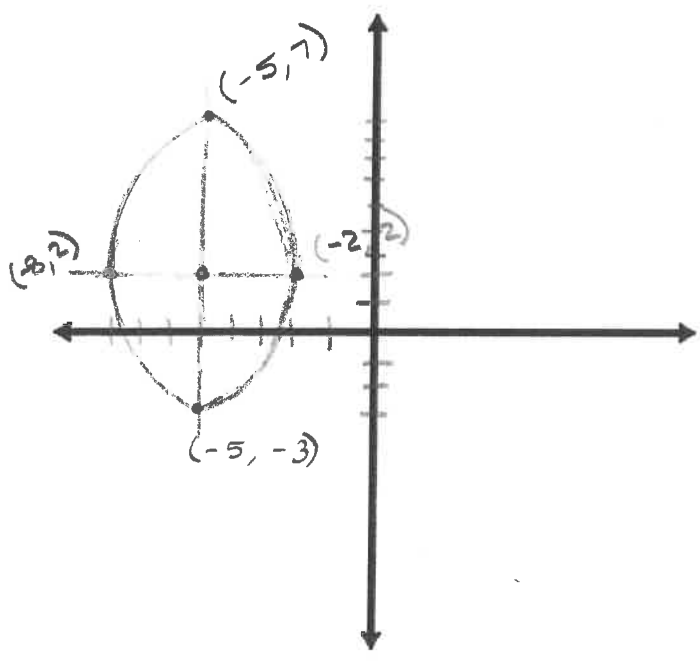
$= 6$

$b = \frac{1}{2}(12-4)$

$= 4$

$$\boxed{\frac{(x-3)^2}{36} + \frac{(y-8)^2}{16} = 1}$$

5. Find the coordinates of the center and foci, and the lengths of the major and minor axes of an ellipse with equation $25x^2 + 9y^2 + 250x - 36y + 436 = 0$. Then graph the ellipse.



$25x^2 + 250x + 9y^2 - 36y = -436$

$25(x^2 + 10x \quad) + 9(y^2 - 4y \quad) = -436$

$25(x^2 + 10x + 25) + 9(y^2 - 4y + 4) = -436$

$+625$

$+36$

$25(x+5)^2 + 9(y-2)^2 = 225$

$$\frac{25(x+5)^2}{225} + \frac{9(y-2)^2}{225} = 1$$

$$\frac{(x+5)^2}{9} + \frac{(y-2)^2}{25} = 1$$

foci

$(-5, -2)$

and $(-5, 6)$

major axis = 10

minor axis = 6

$c^2 = 5^2 - 3^2$

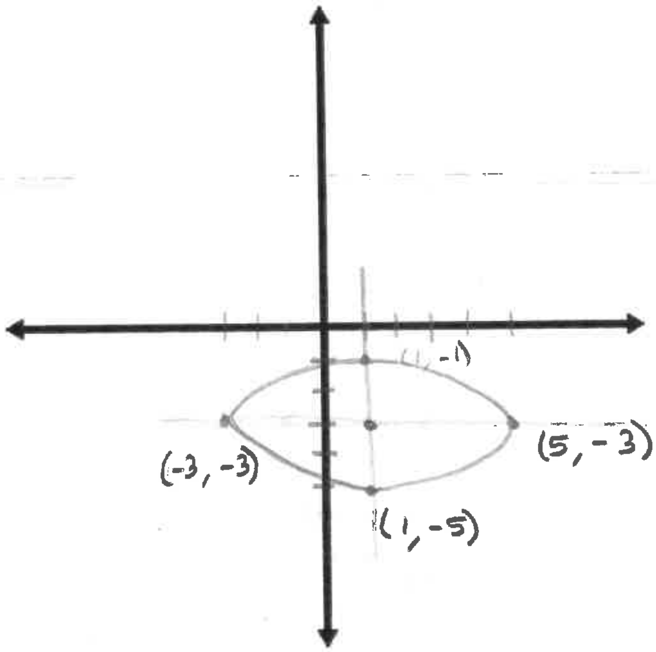
$c = 4$

center (-5, 2)

$a = 5$

$b = 3$

6. Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $x^2 + 4y^2 - 2x + 24y + 21 = 0$. Then graph the ellipse.



$x^2 - 2x + 4y^2 + 24y = -21$

$(x^2 - 2x + 1) + 4(y^2 + 6y + 9) = -21 + 1 + 36$

$(x-1)^2 + 4(y+3)^2 = 16$

$\frac{(x-1)^2}{16} + \frac{(y+3)^2}{4} = 1$

$a = 4$ $b = 2$

$c^2 = 4^2 - 2^2$

$c = \sqrt{12} \approx 3.46$

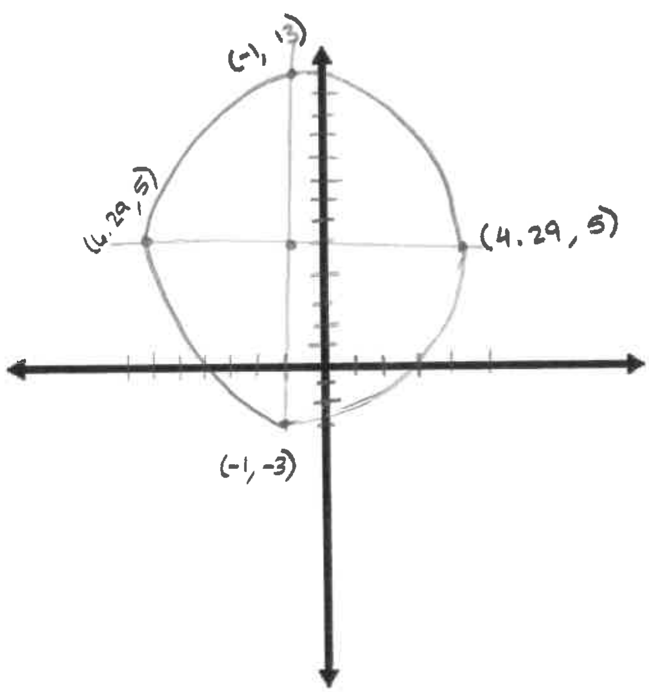
center $(1, -3)$

major axis = 8

minor axis = 4

foci: $(4.46, -3)$ and $(-2.46, -3)$

7. Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $\frac{(y+1)^2}{64} + \frac{(x-5)^2}{28} = 1$. Then graph the ellipse.



center $(-1, 5)$

$a = 8$ $b = \sqrt{28} \approx 5.29$

major axis = 16

minor axis = 10.58

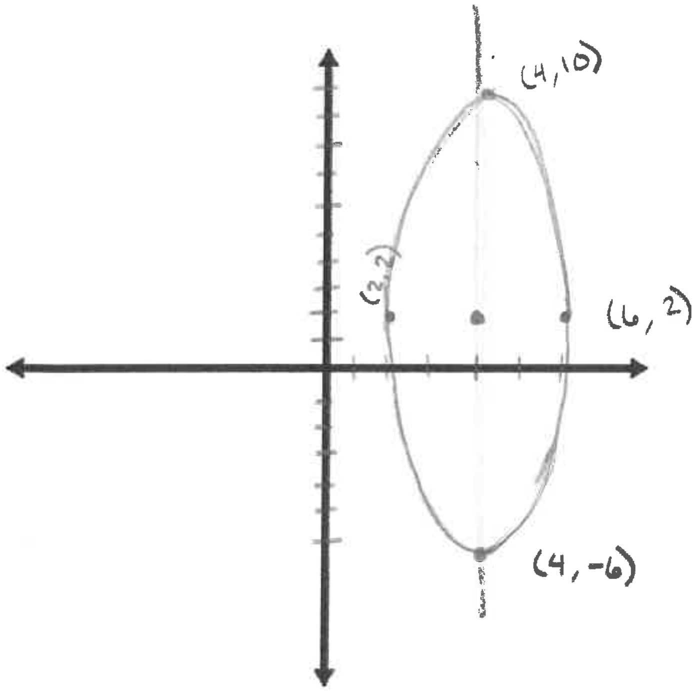
$c^2 = 8^2 - \sqrt{28}^2$

$c = \sqrt{36} = 6$

foci

$(-1, 11)$ and $(-1, -1)$

8. Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $4x^2 + y^2 - 32x - 4y + 52 = 0$. Then graph the ellipse.



$4x^2 - 32x + y^2 - 4y = -52$

$4(x^2 - 8x + 16) + (y^2 - 4y + 4) = -52 + 64 + 4$

$4(x-4)^2 + (y-2)^2 = 16$

$\frac{(x-4)^2}{4} + \frac{(y-2)^2}{16} = 1$

$a = 2$ $\quad b = 4$

center $(4, 2)$

major axis $= 8$

minor axis $= 4$

$c^2 = 8^2 - 4^2$

$c = \sqrt{48} \approx 6.93$

foci

$(4, 8.93)$ and

$(4, -4.93)$