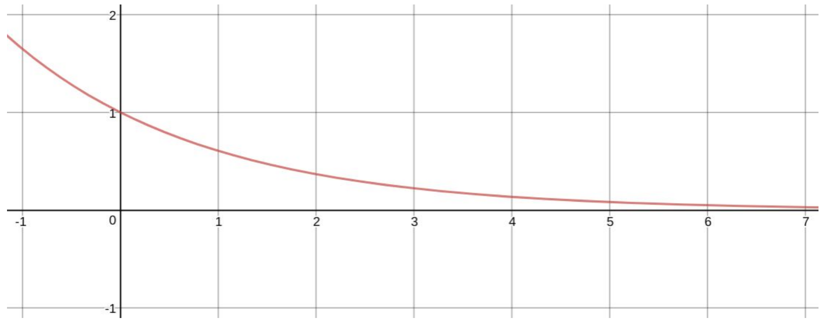
Consider $f(x) = e^{-x/2}$

What is the area under the curve from 0 to ∞?



$$A(b) = \int_0^b e^{-x/2}dx \quad = -2e^{-\frac{x}{2}}\Big|_0^b \quad = -2e^{-b/2} + 2$$

$$\lim_{n\to\infty} A(b) = \lim_{n\to\infty}(-2e^{-b/2} + 2) = 2$$

$$\int_0^\infty e^{-x/2}dx = \lim_{b\to\infty}\int_0^b e^{-x/2}dx = 2$$

Improper Integrals with Infinite Integration Limits

Integrals with infinite limits of integration are **improper integrals**

1. If $f(x)$ is continuous on $[a, \infty)$, then:
$$\int_a^\infty f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$$

2. If $f(x)$ is continuous on $[-\infty, b]$, then:
$$\int_{-\infty}^a f(x)dx = \lim_{a\to\infty}\int_a^b f(x)dx$$

*limit finite the improper integral converges and the limit is the value of the improper integral.
*limit fails to exist the improper integral diverges.

3. If $f(x)$ is continuous on $(-\infty, \infty)$, then:
$$\int_{-\infty}^\infty f(x)dx = \int_{-\infty}^c f(x)dx + \int_c^\infty f(x)dx,$$
where c is any real number

Limit converges if both improper integrals converge, otherwise it diverges

1. Evaluate the improper integral and determine if it converges or diverges:

   a. $\int_{1}^{\infty} \frac{2}{x^3} dx$

   d. $\int_{-\infty}^{\infty} e^x$

   b. $\int_{1}^{\infty} \frac{dx}{\sqrt[4]{x}}$

   e. $\int_{1}^{\infty} \frac{dx}{x}$

   c. $\int_{-\infty}^{0} \frac{dx}{(x-2)^3}$

f. $\int_{0}^{\infty} \frac{2\,dx}{x^2+4x+3}$

g. $\int_{-\infty}^{0} \frac{2\,dx}{x^2-4x+3}$

h. $\int_{1}^{\infty} xe^{-x}\,dx$

i. $\int_0^{\infty} (x+1)e^{-x}\, dx$

j. $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

Integrand with Infinite Discontinuities: integrand has a vertical asymptote!

Improper Integrals with Infinite Discontinuities

Integrals of functions that become infinite at a point within the interval of integration are **improper integrals**

1. If $f(x)$ is continuous on $(a,\ b]$, then:

$$\int_a^b f(x)dx = \lim_{c \to a^+} \int_c^b f(x)dx$$

2. If $f(x)$ is continuous on $[a,\ b)$, then:

$$\int_a^b f(x)dx = \lim_{c \to b^-} \int_a^c f(x)dx$$

*limit finite the improper integral converges and the limit is the value of the improper integral.
*limit fails to exist the improper integral diverges.

3. If $f(x)$ is continuous on $[a,\ c) \cup (c,\ b]$, then:

$$\int_a^c f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Limit converges if both improper integrals converge, otherwise it diverges

2. Evaluate the following integrals and determine whether they converge or diverge:

a. $\int_0^3 \frac{dx}{(x-1)^{2/3}}$

b. $\int_1^2 \frac{dx}{(x-2)}$

Comparison Test

Let f and g be continuous on $[a, \infty)$ with $0 \le f(x) \le g(x)$ for all $x \ge a$. Then:

1. $\int_a^\infty f(x)dx$ converges if $\int_a^\infty g(x)dx$ converges
2. $\int_a^\infty g(x)dx$ diverges if $\int_a^\infty f(x)dx$ diverges

Challenge!

3. Find the volume of the solid obtained by revolving the curve $y = xe^{-x}$, $0 \le x \le \infty$ about the x-axis.