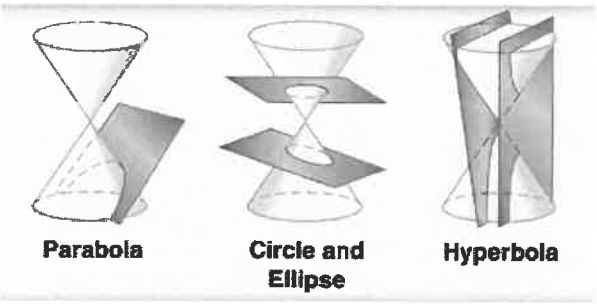
The equation for any conic section can be written in the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

where A, B, and C are not all zero.



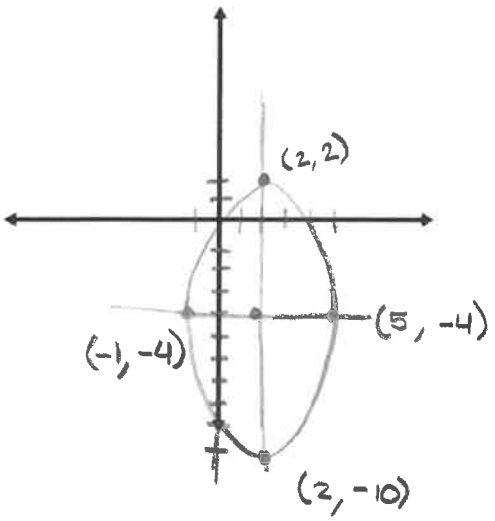
This general form can be converted to standard forms by completing the square

Conic Section	Standard Form of Equations
Circle	$(x-h)^2 + (y-k)^2 = r^2$
Parabola	$y = a(x-h)^2 + k$
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

difference + 1

horizontal axis

1. Write $4x^2 + y^2 - 16x + 8y - 4 = 0$ in standard form. State whether the graph of the equation is a parabola, circle, or ellipse. Then graph the equation.



$4x^2 - 16x + y^2 + 8y = 4$

$4(x^2 - 4x + 4) + (y^2 + 8y + 16) = 4 + 16 + 16$

$4(x-2)^2 + (y+4)^2 = 36$

$\frac{(x-2)^2}{9} + \frac{(y+4)^2}{36} = 1$

3^2 6^2

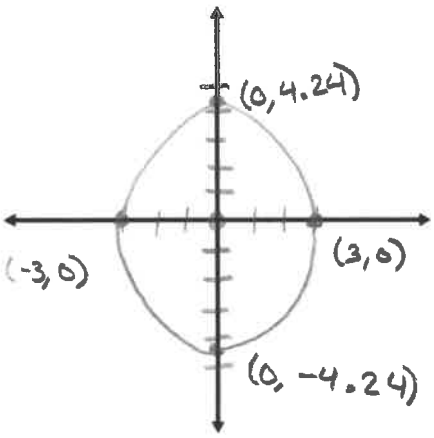
ellipse

major axis = 12

minor axis = 6

center = (2, -4)

2. Write $y^2 = 18 - 2x^2$ in standard form. State whether the graph of the equation is a parabola, circle, or ellipse. Then graph the equation.



$y^2 + 2x^2 = 18$

$2x^2 + y^2 = 18$

$\frac{x^2}{9} + \frac{y^2}{18} = 1$

3^2 $\sqrt{18} \approx 4.24$

ellipse

center (0,0)

major axis = 8.48

minor axis = 6

Can determine the type of conic without writing the equation

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ in standard form

Discriminant $B^2 - 4AC$

Discriminant	Conic Section
$B^2 - 4AC < 0$; $B = 0$ and $A = C$	circle
$B^2 - 4AC < 0$; $B \neq 0$ and $A \neq C$	ellipse
$B^2 - 4AC = 0$	parabola
$B^2 - 4AC > 0$	hyperbola

3. Without writing in standard form, state whether the graph of each equation is a parabola, circle, ellipse, or hyperbola.
   a. $y^2 + 4x^2 - 3xy + 4x - 5y - 8 = 0$

B = -3
A = 4
C = 1

$(-3)^2 - 4(4)(1)$

$9 - 16$

-7

ellipse

b. $3x^2 - 6x + 4y - 5y^2 + 2xy - 4 = 0$

B = 2
A = 3
C = −5

$2^2 - 4(3)(-5)$
$4 + 60$
64 hyperbola

c. $4y^2 - 8x + 6y - 14 = 0$

B = 0
A = 0
C = 4

parabola

d. $y^2 - 2x - 4y + 10 = 0$

B = 0
A = 0
C = 1

$0^2 - 4(0)(1)$
0
parabola

e. $2x^2 + 2y^2 + 16x - 20y = -32$

B = 0
A = 2
C = 2

circle