Vector Functions and Space Curves

Let $\mathbf{r}$ be a **vector function** whose domain is a set of real numbers and result is a three-dimensional vector. Let

$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

where $f(t)$, $g(t)$, and $h(t)$ are real valued functions and are called the component functions of $\mathbf{r}$.

The limit of a vector function $\mathbf{r}$ is defined by taking the limits of its component functions:

$$\lim_{t \to a} \mathbf{r}(t) = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$$

A vector function $\mathbf{r}$ is continuous if and only if its component functions $f(t)$, $g(t)$, and $h(t)$ are continuous.

Example: Given $\mathbf{r}(t) = \left\langle t\sqrt{t+5},\ t^2+2,\ \dfrac{e^t-1}{t} \right\rangle$

a) Find the domain of $\mathbf{r}(t)$.

b) Find all t where $\mathbf{r}(t)$ is continuous.

c) Compute $\lim_{t \to 0} \mathbf{r}(t)$.

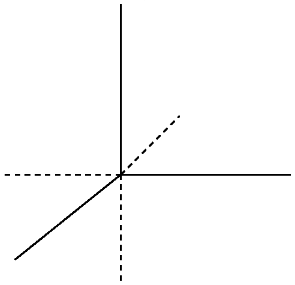
Definition: Suppose that $f(t)$, $g(t)$, and $h(t)$ are real valued functions on an interval I, then the set C defined as :

$$C = \{(x, y, z) | x = f(t), y = g(t), z = h(t)\}$$

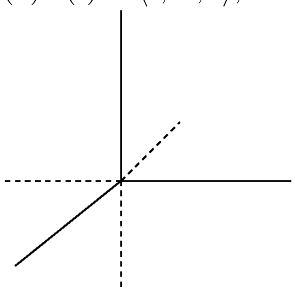
where t is a parameter and t varies in some interval, I, is called a **space curve**. The space curve C can be traversed by the vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$.

Example: Describe the curve defined by the vector function. Indicate the direction of motion.

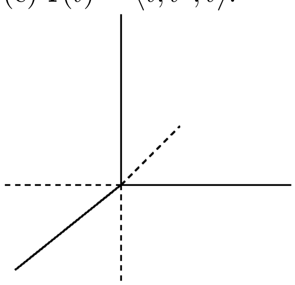
(a) $\mathbf{r}(t) = \langle t, t^2, 0 \rangle$



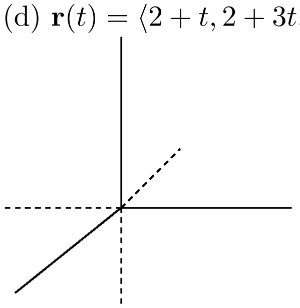
(b) $\mathbf{r}(t) = \langle t, t^2, c \rangle$, where c is a constant.



(c) $\mathbf{r}(t) = \langle t, t^2, t \rangle$.



(d) $\mathbf{r}(t) = \langle 2 + t, 2 + 3t, 4 - 2t \rangle$, $0 \le t \le 1$



Example: Show that the curve $\mathbf{r}(t) = \left\langle \sin(t), 2\cos(t), \sqrt{3}\sin(t) \right\rangle$ lies on both a plane and a sphere. What does the space curve for $\mathbf{r}(t)$ look like?

Example: Find a vector function that represents the curve of intersection of the two surfaces.
$x^2 + y^2 = 4$ and $z = xy$

Example: Sketch the curve $x = \cos^2 t$, $y = \sin^2 t$, and $z = t$.