Big Idea #2 Derivatives

Concept	Question
1. Constructing a Tangent Line	Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$. Write an equation for the line tangent to the curve at the point $(-1, 1)$.
2. Constructing a Normal Line	Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$. Write an equation for the normal line to the curve at the point $(-1, 1)$.
3. Definitions of Slope $\lim\limits_{h \to 0} \frac{f(a+h) - f(a)}{h}$ and $\lim\limits_{x \to a} \frac{f(x) - f(a)}{x - a}$	What is $\lim\limits_{h \to 0} \frac{8(\frac{1}{2}+h)^8 - 8(\frac{1}{2})^8}{h}$
	What is $\lim\limits_{x \to 2} \frac{e^x - e^2}{x - 2}$

<table>
<tr>
<td rowspan="2">4. Basic Derivative</td>
<td>If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$</td>
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<tr>
<td>Let f and g be differentiable functions with the following properties
I. $g(x) > 0$ for all x
II. $f(0) = 1$
If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

A. $f'(x)$
B. $g(x)$
C. e^x
D. 0
E. 1</td>
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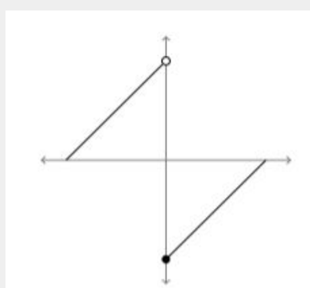
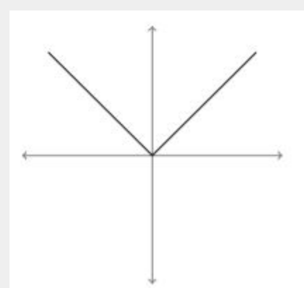
<tr>
<td>5. Graphing the derivative from the original graph</td>
<td>Graph the derivative of the function on the same graph as the function.
 </td>
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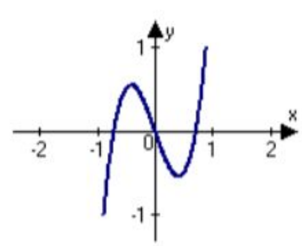
<tr>
<td>6. Graphing the original function from the derivative graph</td>
<td>Below is the graph of the derivative of f. Graph the original function on the same graph.
$y = f'(x)$

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7. One-side Derivatives	$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & ,x \neq 2 \\ 1 & ,x = 2 \end{cases}$$ Let f be the function defined above. Which of the following statements about f are true? I. f has a limit at $x = 2$ II. f is continuous at $x = 2$ III. f is differentiable at $x = 2$
8. Determining if a function is differentiable with limits	$$f(x) = \begin{cases} cx + d & ,x \leq 2 \\ x^2 - cx & ,x > 2 \end{cases}$$ Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?
9. Derivative fails to exist knowing parent graphs *corner, cusp, etc.	Let f be the function given by $f(x) = \|x - 2\|$. Which of the following statements about f are true? I. f is continuous at $x = 2$ II. f is differentiable at $x = 2$ III. f has no absolute minimum at $x = 2$
10. Differentiability implies continuity	The function f is differentiable at $x = 3$. Which of the following statements is guaranteed to be false? I. $\lim_{x \to 3} f(x)$ exists II. $\lim_{h \to 0} \frac{f(3+h)-f(3)}{h}$ exists III. $f(3)$ exists IV. $f''(3)$ exists V. $\lim_{x \to 3} f(x) = f(3)$

11. Product Rule	If $f(x) = (x-1)(x^2+3)^3$, then $f'(x) =$
12. Quotient Rule	What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2-2}{x-1}$?
13. Derivative of all 6 trig functions	If $f(x) = \tan(2x)$, then $f'(\frac{\pi}{6}) =$
	If $f(x) = \sin(e^{-x})$, then $f'(x) =$
	If $g(x) = \cos(x^2)$, then $f'(x) =$
	$\frac{d}{dx}\cot(3x)$
	Find the derivative of $y = 3\sec(\pi x)$

	Find the derivative of $f(x) = \csc(5x^2)$
14. Chain Rule	$\frac{d}{dx}\cos^2(x^3)$
15. Derivative of Inverse Function	<table><tr><td>x</td><td>$f(x)$</td><td>$f^{-1}(x)$</td></tr><tr><td>1</td><td>2</td><td>$\frac{1}{2}$</td></tr><tr><td>2</td><td>3</td><td>$\frac{1}{3}$</td></tr><tr><td>3</td><td>1</td><td>-2</td></tr></table> If f and f^{-1} exist, are continuous and differentiable for $x > 0$, then $\frac{d}{dx}(f^{-1}(x))$ at $x = 1$ is what?
16. Implicit Differentiation	The slope of the line tangent to the curve $y^2 + (xy+1)^3 = 0$ at $(2,-1)$ is what?

17. Derivatives of all 6 inverse trig functions	Find the derivative of $\sin^{-1}(x^2)$
	$\frac{d}{dx}\cos^{-1}(3x)$
	$\frac{d}{dx}\tan^{-1}(e^x)$
	Find the derivative of $\sec^{-1}(x)$
	Find the derivative of $\cot^{-1}(5x^2)$
	$\frac{d}{dx}\csc^{-1}\frac{x}{2}$
18. Derivative of exponential	If $f(x) = \frac{e^{2x}}{2x}$, then $f'(x) =$

	If $f(x) = (x-1)^{\frac{3}{2}} + \frac{e^{x-2}}{2}$, then $f'(2) =$
19. Derivative of logarithmic	If $f(x) = x^2 + 2x$, then $\frac{d}{dx}(f(\ln x)) =$
	If $f(x) = (\ln x)^2$, then $f''(\sqrt{e}) =$
20. L'hospital's Rule	Functions f, g, and h are twice differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{2}{3}(x-2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$. The function h satisfies $h(x) = \frac{x^2-4}{1-(f(x))^3}$ for $x \neq 2$. It is known that $\lim_{x\to 2} h(x)$ can be evaluated using L'hospital's Rule. Use $\lim_{x\to 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

21. Derivatives from a table

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	6	5	3	-2
1	3	-3	-1	2
3	1	-2	2	3

The table above gives values of f, f', g, g' at selected values of x. If $h(x) = f(g(x))$, then $h'(1) =$