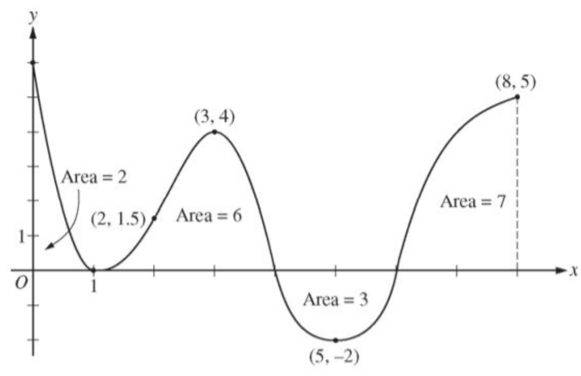
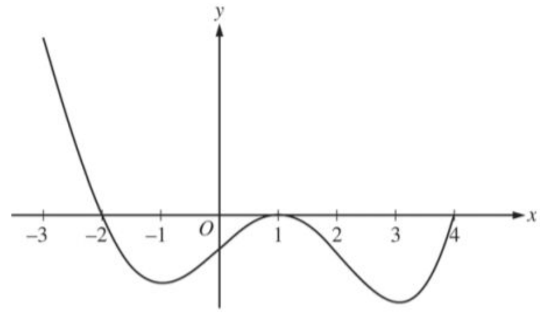
Big Idea #3 Application of Derivatives

Concept	Question
1. Velocity	A particle moves along the $x-$axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?
2.. Acceleration	What is the maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$?
3. Speed	(Calc Active) A particle moves along a straight line. For $0 \leq t \leq 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$. a. Find all times t in the interval $2 \leq t \leq 4$ for which the speed of the particle is 2. b. Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

4. When a particle changes direction	(Calc Active) A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$. Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.
5. Absolute Extrema	 Graph of f' The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$. a. Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

6. Local Extrema	 Graph of f' The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the $x-$axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively. a. Find all $x-$coordinates at which f has a relative maximum. Give a reason for your answer.

7. Mean Value Theorem

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values are given in the table above.

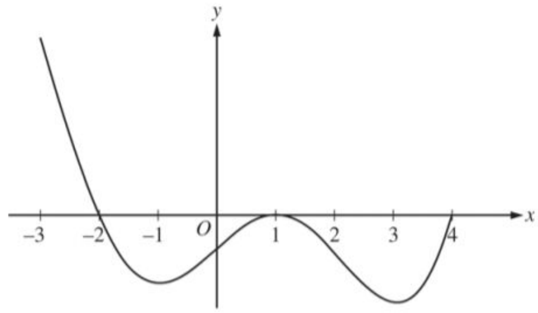
a. Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

8. Function Increasing or Decreasing

What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

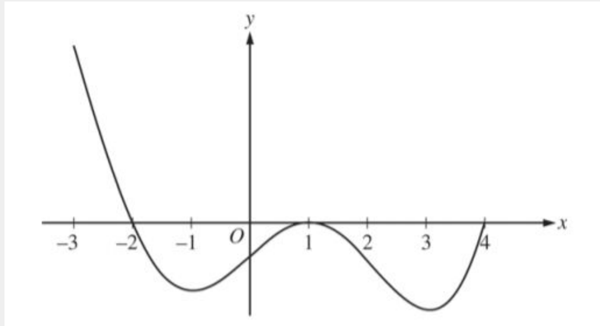
9. First Derivative Test

For what value of x does the function $f(x) = (x-2)(x-3)^2$ have a relative maximum?

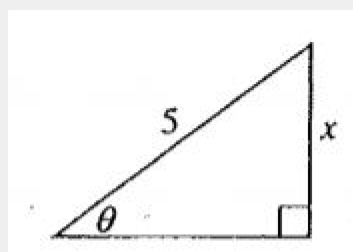
<table>
<tr><td rowspan="2">10. Concavity - concave up or down</td><td>

Graph of f'

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the $x-$axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.
a. On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.</td></tr>
<tr><td>The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for
a. $x < 0$
b. $x > 0$
c. $x < -2$ or $x > -\frac{2}{3}$
d. $x < \frac{2}{3}$ or $x > 2$
e. $\frac{2}{3} < x < 2$</td></tr>
</table>

11. Points of Inflection	Graph of f' The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the $x-$axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively. a. Find the $x-$coordinates for all points of inflection for the graph of f. Give a reason for your answer.
12. Second Derivative Test	What values of x does $f(x) = x^3 - 3x^2 + 12$ have a maximum on the closed interval $[-2, 4]$?

<table>
<tr><td rowspan="2">13. Optimization
*Open Box
*Area of a field and fencing</td><td>A gardener wants to make a rectangular enclosure using a wall as one side and 120 m of fencing for the other three sides. Express the area in terms of x, and find the value of x that gives the greatest area.</td></tr>
<tr><td>A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?</td></tr>
</table>

14. Linear Approximation *using a tangent line to approximate a function value	(Calc Active) Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days. a. For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.
15. Related Rates *Similar Triangles (lamp post) *Sliding Ladder *Cone of water with a leak	 In the triangle shown above, if θ increases at a constant rate of 3 radians per minutes, at what rate is x increasing in units per minute when $x = 3$ units.

Water is draining from a right conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h-12)$ feet per minute. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$)

a. Write an expression for the volume of water in the conical tank as a function of h.

b. At what rate is the volume of water in the conical tank changing when $h = 3$?