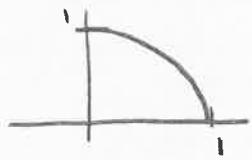
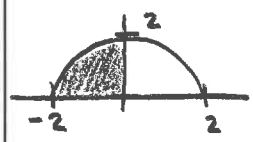
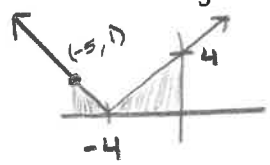
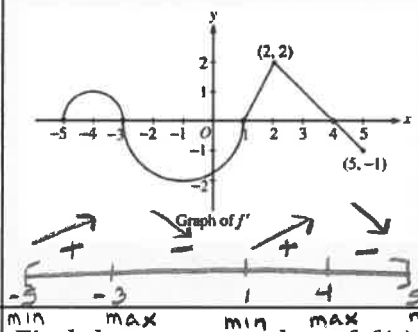
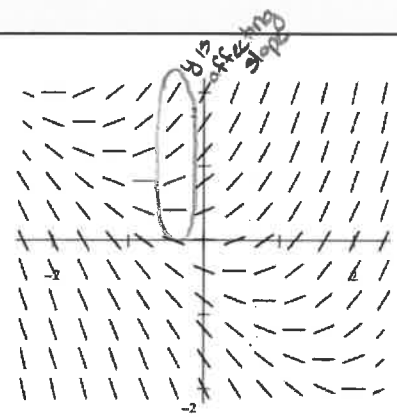
Big Idea #4 Integral and Accumulation

Concept	Question
1. Left Riemann Sum	Find the area under the curve $y = \sqrt{1-x^2}$ from 0 to 1 using a LRAM with 5 rectangles. width = 1/5  $A \approx 1/5\left(y(0) + y(1/5) + y(2/5) + y(3/5) + y(4/5)\right)$ $= 0.8593$
2. Right Riemann Sum	Find the area under the curve $y = \sqrt{1-x^2}$ from 0 to 1 using a RRAM with 5 rectangles. $A \approx 1/5\left(y(1/5) + y(2/5) + y(3/5) + y(4/5) + y(1)\right)$ $= 0.6593$
3. Midpoint Riemann Sum	Find the area under the curve $y = \sqrt{1-x^2}$ from 0 to 1 using a MRAM with 4 rectangles. width = 1/4 intervals: $[0, 1/4]$ $[1/4, 2/4]$ $[2/4, 3/4]$ $[3/4, 4/4]$ $1/8$ $3/8$ $5/8$ $7/8$ $A \approx 1/4\left(y(1/8) + y(3/8) + y(5/8) + y(7/8)\right)$ $= 0.7959$
4. Trapezoidal Sum	$\begin{array}{c\|c\|c\|c\|c\|c} x & -5 & -3 & 0 & 1 & 5 \\ \hline f(x) & 10 & 7 & 5 & 8 & 11 \end{array}$ Given the values for $f(x)$ on the table above, approximate the area under the graph of $f(x)$ from $x = -5$ to $x = 5$ using four subintervals and a **Trapezoidal approximation.** $A \approx \frac{1}{2}(2)(10+7) + \frac{1}{2}(3)(7+5) + \frac{1}{2}(1)(5+8) + \frac{1}{2}(4)(8+11)$ $= 79.5$

5. Properties of Integrals	Given $\int_0^5 f(x)dx = 10$ and $\int_5^7 f(x)dx = 3$, find a) $\int_0^7 f(x)dx = 13$ b) $\int_5^0 f(x)dx = -10$ c) $\int_5^5 f(x)dx = 0$ d) $\int_0^5 3f(x)dx = 30$
6. Calculating Integrals Using Geometry	Find the following: a. $\int_{-2}^{0} \sqrt{4-x^2}dx = \pi$  $= \frac{1}{4}\pi(2)^2 = \pi$ b. $\int_{-5}^{0} \lvert x+4\rvert dx = 8.5$  $= \frac{1}{2}(1)(1) + \frac{1}{2}(4)(4)$ $= 1/2 + 8$ $= 8.5$
7. Basic Antiderivative	$\int \frac{1}{3}x^4 dx$ $= \frac{1}{3}\frac{x^5}{5} + C = \frac{1}{15}x^5 + C$
8. Trig Antiderivative	$\int \cos x dx = \sin x + C$ $\int \sec x \tan x dx = \sec x + C$ $\int \sin x dx = -\cos x + C$ $\int \csc^2 x dx = -\cot x + C$ $\int \sec^2 x dx = \tan x + C$ $\int \csc x \cot x dx = -\csc x + C$
9. Inverse Trig Antiderivative	$\int \frac{1}{x^2+1}dx = \tan^{-1}x + C$ $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + C$ $\int \frac{1}{\lvert x\rvert\sqrt{x^2-1}}dx = \sec^{-1}x + C$
10. Fundamental Theorem of Calculus Part 1	Find $\frac{d}{dx}\int_2^{x^2} \cos(t)dt = 2x\cos x^2$

11. Fundamental Theorem of Calculus Part 2 abs. min is 3 @ $x=1$ b/c $f'(x)$ changes from neg to pos @ $x=1$ & $f(3) < f(x)$ for all $-5 \le x \le 5$	Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f' the derivative of f, consists of two semicircles and two line segments, as shown below. Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.  ★ $\int_1^x f'(t)\,dt = f(x) - f(1)$ ★ $\begin{array}{c\|c} x & f(x) \\ \hline -5 & 3+\frac{3}{2}\pi \\ 1 & 3 \\ 5 & 5.5 \end{array}$ see attached paper for work
12. Average Mean Value	Find the average value of $f(x) = x^2 + 1$ from -1 to 3. $\frac{1}{3-(-1)}\int_{-1}^{3}(x^2+1)\,dx = \frac{1}{4}\left(\frac{1}{3}x^3 + x\right)\Big\|_{-1}^{3}$ $= \frac{1}{4}\left[\left(\frac{1}{3}(27)+3\right) - \left(-\frac{1}{3}-1\right)\right]$ $= \boxed{10/3}$
13. U-sub	$\int_0^{\pi} \cos\sqrt{\sin x}\,dx$ $u = \sin x$ $du = \cos x\,dx$ $\int_0^0 u^{1/2}\,du$ $= \boxed{0}$
14. Slope Field	Quad II x neg slope pos & neg  Shown above is the slope field for which of the following differential equations? ~~(A)~~ $\frac{dy}{dx} = 1 + x$ ~~(B)~~ $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ ~~(D)~~ $\frac{dy}{dx} = \frac{x}{y}$ ~~(E)~~ $\frac{dy}{dx} = \ln y$ y would not be able to be neg or 0

15. Differential Equations	The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then $$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$ Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$. (c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$. $\int \frac{1}{100-B}\,dB = \int \frac{1}{5}\,dt$ $-\ln\lvert 100-B\rvert = \frac{1}{5}t + C$ $-\ln\lvert 100-20\rvert = \frac{1}{5}(0) + C$ $-\ln 80 = C$ $-\ln\lvert 100-B\rvert = \frac{1}{5}t - \ln 80$ $\ln\lvert 100-B\rvert = \ln 80 - \frac{1}{5}t$ $100 - B = e^{\ln 80 - \frac{1}{5}t}$ $-B = 80e^{-\frac{1}{5}t} - 100$ $\boxed{B = 100 - 80e^{-\frac{1}{5}t}}$
16. Exponential Growth/Decay	The half-life of the radium isotope Ra-226 is approximately 1,599 years. If the initial quantity of the isotope is 38 g, what is the amount left after 1,000 years? Round your answer to two decimal places. a. 24.63 g b. 30.60 g c. 25.13 g d. 11.88 g e. 12.32 g (1599, 19) * half orginal $y = 38e^{kt}$ $19 = 38e^{k(1599)}$ $\ln \frac{1}{2} = 1599k$ $\frac{\ln \frac{1}{2}}{1599} = k$ $y = 38e^{t\frac{\ln \frac{1}{2}}{1599}}$ $y = 38e^{\frac{1000 \ln \frac{1}{2}}{1599}}$ $= 24.63$

Work for #11

$$\int_{-5}^{1} f'(x)\,dx = f(1) - f(-5)$$

$$f(-5) = f(1) - \int_{-5}^{1} f'(x)\,dx$$

$$= 3 - \left[\tfrac{1}{2}\pi(1)^2 - \tfrac{1}{2}\pi(2)^2\right]$$

$$= 3 - \left(\tfrac{1}{2}\pi - 2\pi\right)$$

$$= 3 + \tfrac{3}{2}\pi$$

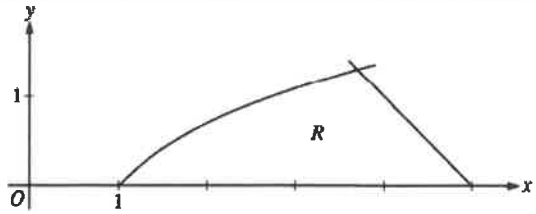
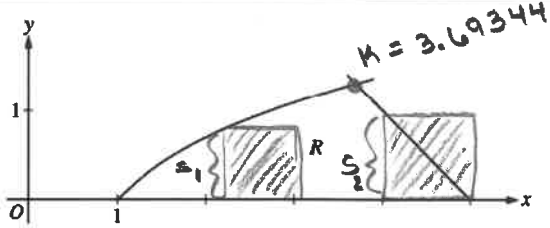
$$\int_{1}^{5} f'(x)\,dx = f(5) - f(1)$$

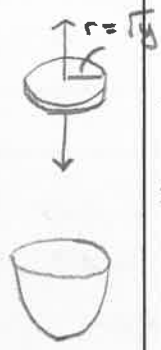
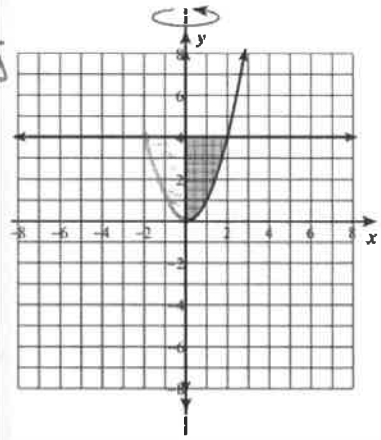
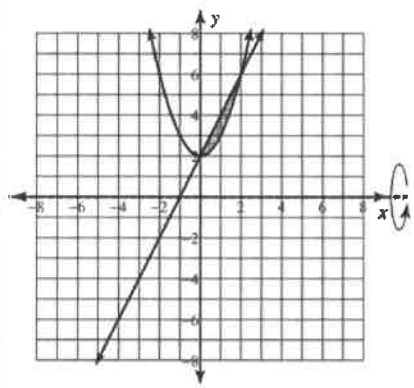
$$f(5) = f(1) + \int_{1}^{5} f'(x)\,dx$$

$$f(5) = 3 + \left[\tfrac{1}{2}(4)(2) - \tfrac{1}{2}(1)(1)\right]$$

$$= 3 + 3 - \tfrac{1}{2}$$

$$= 5.5$$

17. Area Between Curves	 Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above. (a) Find the area of R. $\ln x = 5 - x$ $x \approx 3.69344135 = K$ $\int_1^K \ln x\,dx + \int_K^5 (5-x)\,dx$ $\approx \boxed{2.9858}$
18. Volume → Cross Sections	 Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above. (b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid. $s_1 = \ln x \qquad s_2 = 5 - x$ $A_1 = (\ln x)^2 \qquad A_2 = (5-x)^2$ $V = \int_1^K (\ln x)^2\,dx + \int_K^5 (5-x)^2\,dx$ $\approx \boxed{2.784}$

19. Volume → Disks 	$x=\sqrt{y},\ x=0,\ y=4$  $V=\int_0^4 \pi(\sqrt{y})^2\,dy$ $\approx \boxed{8\pi}$
20. Volume → Washers	$y=2x+2$ $y=x^2+2$ $R=2x+2 \quad r=x^2+2$  $V=\int_0^2 \pi(2x+2)^2\,dx-\int_0^2 \pi(x^2+2)^2\,dx$ $=\boxed{\frac{48\pi}{5}}$ ≈ 30.159
21. L'hospital's Rule	$\lim_{x\to 0}\frac{\sec x-1}{\sin x}$ $\lim_{x\to 0} \sec x-1=0$ $\lim_{x\to 0} \sin x=0$ $\lim_{x\to 0}\frac{\sec x\tan x}{\cos x}=\lim_{x\to 0}\frac{\frac{1}{\cos x}\,\frac{\sin x}{\cos x}}{\cos x}$ $=\lim_{x\to 0}\frac{\sin x}{\cos^3 x}=\frac{0}{1}=\boxed{0}$