

Challenge Problems

1. Find a function $f(x)$ that meets the following criteria:

- $f(3) = 0$

$$f(x) = \frac{2(x-3)(\quad)}{(x-4)(x+4)}$$

- f has a horizontal asymptote at $y = 2$
- f has a vertical asymptote at $x = 4$ and $x = -4$
- $f(0) = 1$ (Meeting this requirement is the trickiest part!)

$$\frac{2(x-3)(x + \frac{2}{3})}{(x-4)(x+4)}$$

$-6 \leftarrow$ need $\frac{2}{3}$ times -6

$$\frac{-6}{-16}$$

$$\frac{2(x-3)(3x+8)}{3(x-4)(x+4)}$$

$$f(x) = \frac{2(x-3)(3x+8)}{3(x-4)(x+4)}$$

2. Factor the polynomial $h(x) = x^5 + 9x^3 - 8x^2 - 72$ into linear and irreducible quadratic factors with real coefficients. Then factor $h(x)$ into linear factors with complex coefficients.

a. Linear and Irreducible quadratic factors of $h(x)$ with real coefficients:

$$\begin{array}{r} 2 \\ \hline 1 \\ 2 \\ \hline 1 \end{array}$$

$$x^5 + 9x^3 - 8x^2 - 72$$

$$x^3(x^2 + 9) - 8(x^2 + 9)$$

$$(x^3 - 8)(x^2 + 9)$$

$$(x-2)(x^4 + 2x^3 + 13x^2 + 16x + 36)$$

would get stuck here
trying to do synthetic division

$$h(x) = (x^3 - 8)(x^2 + 9)$$

$$= (x-2)(x^2 + 2x + 4)(x^2 + 9)$$

b. Linear factors of $h(x)$ with complex coefficients:

$$h(x) = (x-2)(x^2 + 2x + 4)(x + 3i)(x - 3i)$$

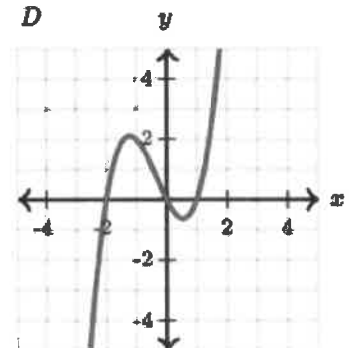
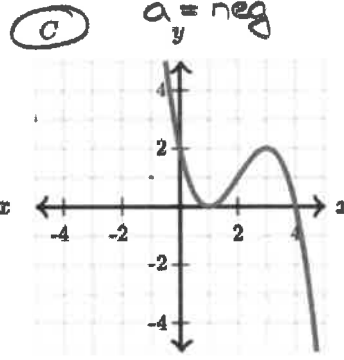
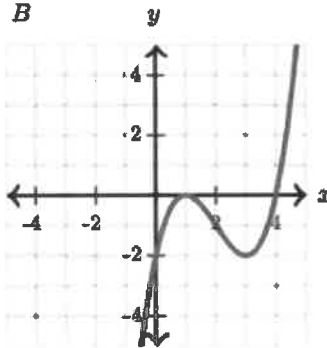
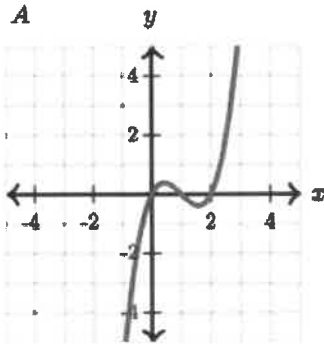
$$x = \frac{-2 \pm \sqrt{4 - 4(1)(4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 - 16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2} = \frac{-2 \pm 2i\sqrt{3}}{2} = -1 \pm i\sqrt{3}$$

$$h(x) = (x-2)[x - (-1 + i\sqrt{3})][x - (-1 - i\sqrt{3})](x + 3i)(x - 3i)$$

3. Which of the following could be the graph of $y = ax^3 + bc^2 + cx + 2$, where a , b , and c are real numbers? (Could be more than one of the graphs below) $y\text{-int} = 2$



4. Which of the following could be the graph of $y = k(x-2)^m(x+1)^n$, where k is a real number, m is an even integer, and n is an odd integer? (Could be more than one of the graphs below) $m+n = \text{odd}$

