1. What is the difference between a limit and a function value?

THEOREM 1 Properties of Limits

If L, M, c, and k are real numbers and

$$\lim_{x\to c} f(x) = L \quad \text{and} \quad \lim_{x\to c} g(x) = M, \text{ then}$$

1. *Sum Rule:* $\lim_{x\to c} (f(x) + g(x)) = L + M$

   The limit of the sum of two functions is the sum of their limits.

2. *Difference Rule:* $\lim_{x\to c} (f(x) - g(x)) = L - M$

   The limit of the difference of two functions is the difference of their limits.

3. *Product Rule:* $\lim_{x\to c} (f(x) \cdot g(x)) = L \cdot M$

   The limit of a product of two functions is the product of their limits.

4. *Constant Multiple Rule:* $\lim_{x\to c} (k \cdot f(x)) = k \cdot L$

   The limit of a constant times a function is the constant times the limit of the function.

5. *Quotient Rule:* $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{L}{M},\ M \neq 0$

   The limit of a quotient of two functions is the quotient of their limits, provided the limit of the denominator is not zero.

6. *Power Rule:* If r and s are integers, $s \neq 0$, then

   $$\lim_{x\to c} (f(x))^{r/s} = L^{r/s}$$

   provided that $L^{r/s}$ is a real number.

   The limit of a rational power of a function is that power of the limit of the function, provided the latter is a real number.

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x\to 0} \frac{1-\cos x}{x} = 0 \qquad \lim_{x\to 0} \frac{\tan x}{x} = 1$$

$$\lim_{x\to 0} \frac{\sin ax}{bx} = \frac{a}{b} \qquad \lim_{x\to 0} \frac{1-\cos ax}{bx} = 0 \qquad \lim_{x\to 0} \frac{\tan ax}{bx} = \frac{a}{b}$$

2. $\lim_{x \to 5^+} \sqrt{x-5} =$

3. $\lim_{x \to 5^-} \sqrt{x-5} =$

4. $\lim_{x \to 5} \sqrt{x-5} =$

5. Use the piecewise function $f(x)=\begin{cases} 3-x & x<-3 \\ 2x+12 & -3\le x<4 \\ 9 & x\ge 4 \end{cases}$ for the limits below:

a. $\lim_{x \to -3^+} f(x) =$

d. $\lim_{x \to 4^-} f(x) =$

g. $\lim_{x \to 7^+} f(x) =$

b. $\lim_{x \to 4^+} f(x) =$

e. $\lim_{x \to -3} f(x) =$

h. $\lim_{x \to -5} f(x) =$

c. $\lim_{x \to -3^-} f(x) =$

f. $\lim_{x \to 4} f(x) =$

i. $\lim_{x \to 2} f(x) =$

A function $f(x)$ has a limit as x approaches c if and only if the right-hand and left-hand limits at c exist and are equal.

$$\lim_{x \to C} f(x) = L \Leftrightarrow \lim_{x \to C^+} f(x) = L \text{ and } \lim_{x \to C^-} f(x) = L$$

3. Find the limits below using the graph:

a. $f(1) =$

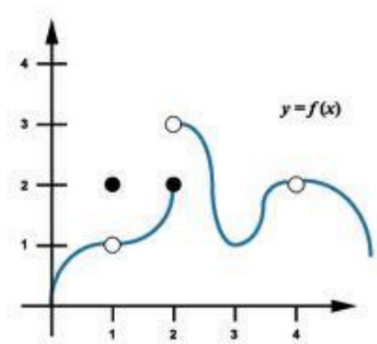
b. $\lim_{x \to 1} f(x) =$

c. $\lim_{x \to 2^+} f(x) =$

d. $\lim_{x \to 2^-} f(x) =$

e. $\lim_{x \to 4} f(x) =$

f. $f(4) =$

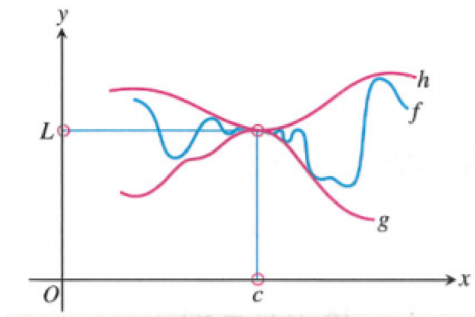


Sandwich Theorem
If $g(x) \le f(x) \le h(x)$ for all $x \ne c$in some interval about c, and

$$\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$$

then

$$\lim_{x\to c} f(x) = L$$



17. Show that $\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$

Horizontal Asymptote:

The line $y = b$ is a horizontal asymptote of the graph of a function $y = f(x)$ if either
$\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$

"Rules"

1) If the denominator is a larger degree than the numerator the limit = ______________
2) If the numerator is a larger degree than the denominator the limit = ______________
3) If the numerator and denominator have the same degree than the limit = __________

a. $\lim_{x\to\infty} \frac{2x^2+3}{5x^2-7} =$

b. $\lim_{x\to\infty} \frac{6x}{3x^2+1} =$

c. $\lim\limits_{x\to-\infty} \dfrac{4x^2}{8-3x^2} =$

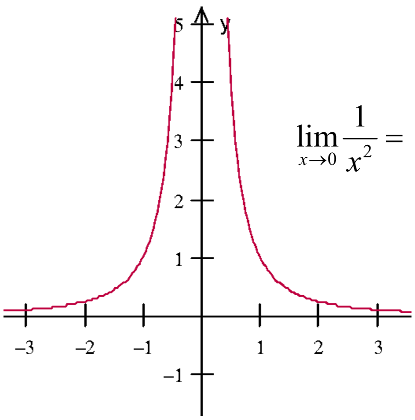
d. $\lim\limits_{x\to-\infty} \dfrac{x^2}{3x-1} =$

Vertical Asymptote:

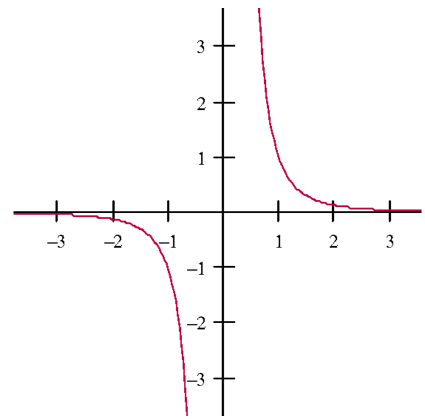
The line $x = a$ is a vertical asymptote of the graph of a function $y = f(x)$ if either

$$\lim_{x\to a^+} f(x) = \pm\infty \text{ or } \lim_{x\to a^-} f(x) = \pm\infty$$

Unbounded Behavior

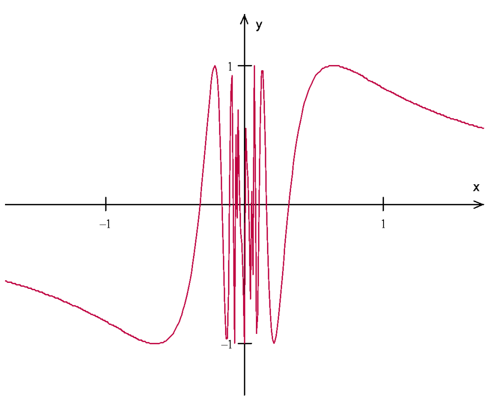


$\lim\limits_{x\to 0} \dfrac{1}{x^2} =$

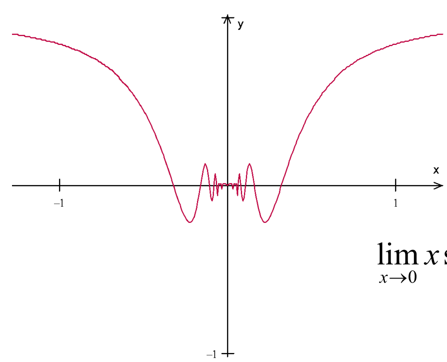


$\lim\limits_{x\to 0} \dfrac{1}{x^3} =$

3) Oscillating Behavior



*Exception to Oscillating Behavior



$\lim\limits_{x\to 0} x \sin\dfrac{1}{x} =$

1. $\lim\limits_{x\to 7^+} \dfrac{1}{x-7} =$

2. $\lim\limits_{x\to 7^-} \dfrac{1}{x-7} =$

3. $\lim\limits_{x\to 7} \dfrac{1}{x-7} =$

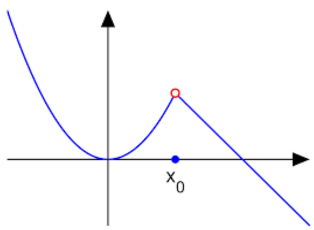
4. $\lim\limits_{x\to 3} \dfrac{2}{(x-3)^2} =$

5. $\lim\limits_{x\to 8} \dfrac{-6}{(x-8)^2} =$

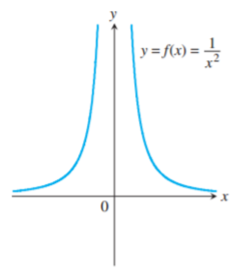
6. $\lim\limits_{x\to \frac{\pi}{2}^+} \tan x =$

Continuity
A function is continuous if:

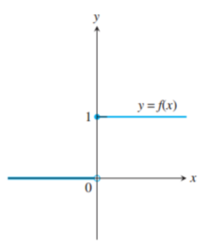
1. Discontinuity:



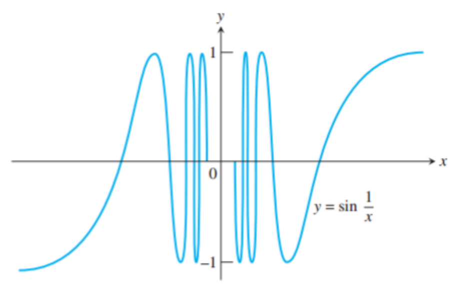
3. Discontinuity:



2. Discontinuity:

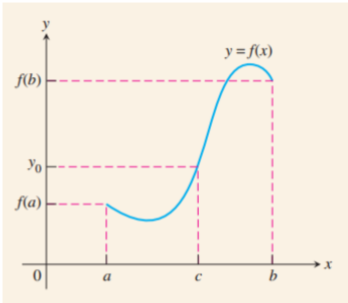


4. Discontinuity:



2. Find the points of continuity and the points of discontinuity of the function. Identify each type of discontinuity.
   a. $y = \frac{x+1}{x^2-4x+3}$

Intermediate Value Theorem
A function $y = f(x)$ that is continuous on a closed interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$. In other words if y_0 is between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$.



2. Find each point of discontinuity for the function below. Then if there are any, determine if the discontinuities are removable.

$$f(x)=\begin{cases}-2x, & x\le 2\\ x^2-4x+1, & x>2\end{cases},$$

3. What value should be assigned to k to make f a continuous function?

$$f(x)=\begin{cases}\dfrac{x^2+2x-15}{x-3}, & x\neq 3\\ k, & x=3\end{cases}$$

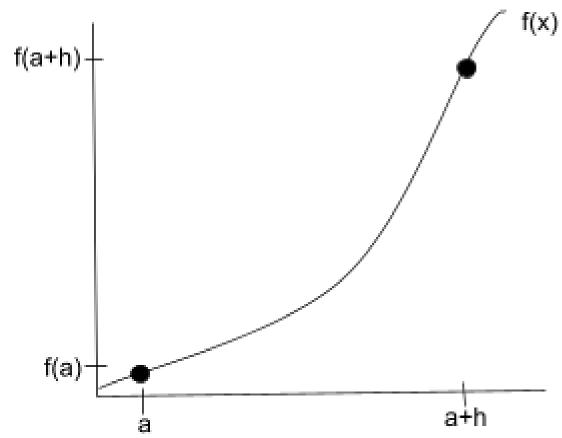
Slope of the tangent Line to a Curve at a Point

The slope of the tangent line to the graph of a function $y = f(x)$ at $(a,\ f(a))$ is given by:

$$m_{tan} = \lim_{h \to 0} \text{\_\_\_\_\_\_\_\_\_\_\_\_}$$

Provided that this limit exists. This limit describes:

- The slope of the graph of f at $(a,\ f(a))$
- The instantaneous rate of change of f with respect to x at a



1. Find the derivative of the following:
   a. $\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

Average Rate of Change:

The average rate of change of f from $x = a$ to $x = a + h$ is given by the difference quotient:

$$\frac{f(a+h) - f(a)}{h}$$

Instantaneous Rate of Change:

The instantaneous rate of change of f with respect to x at a is the derivative of f at a

$$\lim_{x \to a} \frac{f(a+h) - f(a)}{h}$$