If $y=\ln \left(x^{2}+y^{2}\right)$, then the value of $\frac{d y}{d x}$ at the point $(1,0)$ is
(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) 2
(E) undefined

1985 AB 13
If $x^{2}+x y+y^{3}=0$, then, in terms of $x$ and $y, \frac{d y}{d x}=$
(A) $-\frac{2 x+y}{x+3 y^{2}}$
(B) $-\frac{x+3 y^{2}}{2 x+y}$
(C) $\frac{-2 x}{1+3 y^{2}}$
(D) $\frac{-2 x}{x+3 y^{2}}$
(E) $-\frac{2 x+y}{x+3 y^{2}-1}$
$\frac{d}{d x}\left(2^{x}\right)=$
(A) $2^{x-1}$
(B) $\quad\left(2^{x-1}\right) x$
(C) $\left(2^{x}\right) \ln 2$
(D) $\left(2^{x-1}\right) \ln 2$
(E) $\frac{2 x}{\ln 2}$

What is $\lim _{h \rightarrow 0} \frac{8\left(\frac{1}{2}+h\right)^{8}-8\left(\frac{1}{2}\right)^{8}}{h}$ ?
(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) The limit does not exist.
(E) It cannot be determined from the information given.

The $\lim _{h \rightarrow 0} \frac{\tan 3(x+h)-\tan 3 x}{h}$ is
(A) 0
(B) $3 \sec ^{2}(3 x)$
(C) $\sec ^{2}(3 x)$
(D) $3 \cot (3 x)$
(E) nonexistent

$$
\text { If } y=e^{n x}, \text { then } \frac{d^{n} y}{d x^{n}}=
$$

(A) $n^{n} e^{n x}$
(B) $n!e^{n x}$
(C) $n e^{n x}$
(D) $n^{n} e^{x}$
(E) $n!e^{x}$


Graph of $f$

The graph of the piecewise-defined function $f$ is shown in the figure above. The graph has a vertical tangent line at $x=-2$ and horizontal tangent lines at $x=-3$ and $x=-1$. What are all values of $x,-4<x<3$. at which $f$ is continuous but not differentiable?
(A) $x=1$
(B) $x=-2$ and $x=0$
(C) $x=-2$ and $x=1$
(D) $x=0$ and $x=1$


The graph of the derivative of $f$ is shown in the figure above. Which of the following could be the graph of $f$ ?
(A)

(B)

(C)

(D)

(E)



The graph of $f$ is shown in the figure above. Which of the following could be the graph of the derivative of $f$ ?
(A)

(B)

(C)

(D)

(E)


$$
f(x)= \begin{cases}\frac{x^{2}-4}{x-2} & \text { if } x \neq 2 \\ 1 & \text { if } x=2\end{cases}
$$

Let $f$ be the function defined above. Which of the following statements about $f$ are true?
I. $f$ has a limit at $x=2$.
II. $f$ is continuous at $x=2$.
III. $f$ is differentiable at $x=2$.
(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

1969 BC 20
An equation for a tangent to the graph of $y=\arcsin \frac{x}{2}$ at the origin is
(A) $x-2 y=0$
(B) $x-y=0$
(C) $x=0$
(D) $y=0$
(E) $\pi x-2 y=0$

The slope of the line normal to the graph of $y=2 \ln (\sec x)$ at $x=\frac{\pi}{4}$ is
(A) -2
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 2
(E) nonexistent

1993 BC 17
The slope of the line tangent to the graph of $\ln (x y)=x$ at the point where $x=1$ is
(A) 0
(B) 1
(C) $e$
(D) $e^{2}$
(E) $1-e$


The graph of the derivative of $f$ is shown in the figure above. Which of the following could be the graph of $f$ ?
(A)

(B)

(C)

(D)

(E)


1998 BC 2
In the $x y$-plane, the graph of the parametric equations $x=5 t+2$ and $y=3 t$, for $-3 \leq t \leq 3$, is a line segment with slope
(A) $\frac{3}{5}$
(B) $\frac{5}{3}$
(C) 3
(D) 5
(E) 13


The graph of $y=h(x)$ is shown above. Which of the following could be the graph of $y=h^{\prime}(x)$ ?
(A)

(B)

(C)

(D)

(E)


1998 BC 40
Let $f$ and $g$ be functions that are differentiable everywhere. If $g$ is the inverse function of $f$ and if $g(-2)=5$ and $f^{\prime}(5)=-\frac{1}{2}$, then $g^{\prime}(-2)=$
(A) 2
(B) $\frac{1}{2}$
(C) $\frac{1}{5}$
(D) $-\frac{1}{5}$
(E) -2

For $0<x<\frac{\pi}{2}$, if $y=(\sin x)^{x}$, then $\frac{d y}{d x}$ is
(A) $x \ln (\sin x)$
(B) $(\sin x)^{x} \cot x$
(C) $x(\sin x)^{x-1}(\cos x)$
(D) $(\sin x)^{x}(x \cos x+\sin x)$
(E) $(\sin x)^{x}(x \cot x+\ln (\sin x))$

1. Find the values of $a$ and $b$ that will make $f(x)$ differentiable at $x=-1$.

$$
f(x)= \begin{cases}a x^{2}+b x-3, & x<-1 \\ 2 x^{3}-5, & x \geq-1\end{cases}
$$

2. Write an equation for the tangent line to $y=x \cos x$ at $x=\frac{\pi}{2}$.
3. Write an equation for the normal line at $x=0$ to $y=2+e^{-2 x}$.
4. If the line $y=4 x-18$ is tangent to the curve $y=a x^{2}+b x$ at the point $(3,-6)$, then find $a$ and $b$.
5. Find $y=a x^{2}+b x+c$ such that $f(0)=5, f^{\prime}(0)=6$, and $f^{\prime \prime}(0)=-3$.
6. The position (in meters) of an object at any time $t$ (in minutes) is given by the function $s(t)=3 t^{2}-\cos 2 t$.
a. Find the velocity of the object at time $t=\pi$ using appropriate units.
b. Find the acceleration of the object at time $t=\pi$ using appropriate units.
7. Use the table of values below representing the position of an object at the given times.

| $t(\mathrm{sec})$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $s(t)(\mathrm{cm})$ | 2.3 | 5.6 | 6.2 | 6.4 | 4.8 |

a. Find the average velocity of the object between times $t=1$ and $t=4$. Show your computation.
b. Find an estimate for the velocity of the object at $t=3$.
8. Find $\frac{d^{2} y}{d x^{2}}$, for the function $y=2 x^{4}-5 \sqrt{x}$.
9. Find $\lim _{h \rightarrow 0} \frac{\cos \left(\frac{\pi}{3}+h\right)-\frac{1}{2}}{h}$
10. Find $\lim _{h \rightarrow 0} \frac{3(2+h)^{3}-24}{h}$
11. Use the table below to find the specified derivatives.

| $x$ | $f(x)$ | $f^{\prime}(x)$ | $g(x)$ | $g^{\prime}(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | $\frac{1}{3}$ | -2 | -3 |
| 2 | 3 | $\frac{1}{2}$ | 4 | 0 |
| 3 | 1 | -2 | 5 | -1 |

a. If $h(x)=f(x) * g(x)$, find $h^{\prime}(2)$
d. If $h(x)=\frac{2 f(x)}{x^{3}}$, find $h^{\prime}(2)$
b. If $h(x)=\frac{f(x)}{g(x)}$, find $h^{\prime}(3)$
e. If $h(x)=g(f(x))$, find $h^{\prime}(3)$
c. If $h(x)=x^{3} * g(x)$, find $h^{\prime}(1)$
f. If $h(x)=f\left(x^{2}\right)$, find $h^{\prime}(1)$
g. If $h(x)$ is the inverse of $f(x)$, find $h^{\prime}(1)$
12. Find the $78^{\text {th }}$ derivative of $f(x)=3^{x}$
13. Find the $95^{\text {th }}$ derivative of $f(x)=\sin (3 x)$
14. Find the derivative of the function $f(x)=\tan ^{-1}\left(3 x^{2}\right)$
15. Find the derivative of $f(x)=\sin ^{-1}(\cos (3 x))$
16. Find the derivative of the inverse of the function $f(x)=3 x^{5}-2 x^{3}-4$ at $x=-5$.
17. Find the derivative of the function $y=x^{\cos x}$.
18. Which of the following are asymptotes of $2 y+x y-x+3=0$
I. $x=3$
II. $x=-2$
III. $y=1$
a. I only
b. III only
c. I and II only
d. II and III only
e. I, II, and III

