The derivative of the function f with respect to the variable $x$is the function f' whose value at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

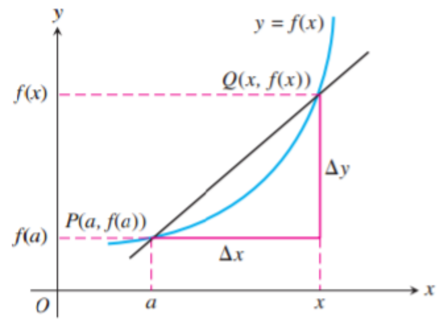
provided the limit exists

Alternative Derivative (derivative at a point)

The derivative of the function f at the point $x = a$ is the limit

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

provided the limit exists.



a. $\lim_{h \to 0} \frac{\cos(x+h)-\cos x}{h}$

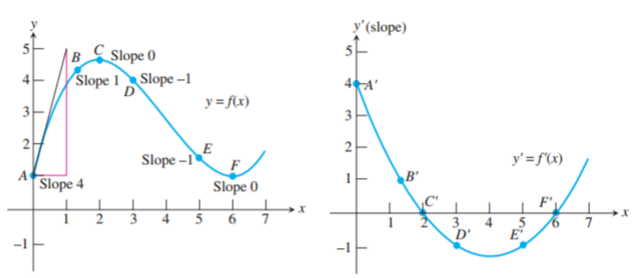
d. $\lim_{h \to 0} \frac{(3+h)^4-81}{h}$

b. $\lim_{h \to 0} \frac{5(x+h)^4-5x^4}{h}$

e. $\lim_{h \to 0} \frac{\sec(x+h)-\sec x}{h}$

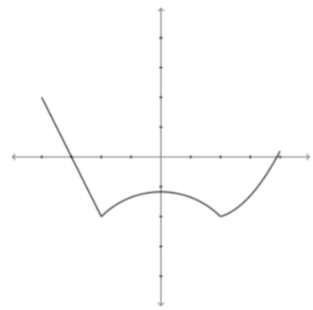
c. $\lim_{h \to 0} \frac{10(2+h)^3-80}{h}$

f. $\lim_{h \to 0} \frac{\sin(\frac{\pi}{6}+h)-\frac{1}{2}}{h}$



(a) (b)

1. Consider the graph of $f(x)$ below to answer the following questions:



   a. Are there any x values for which the derivative does not exist?

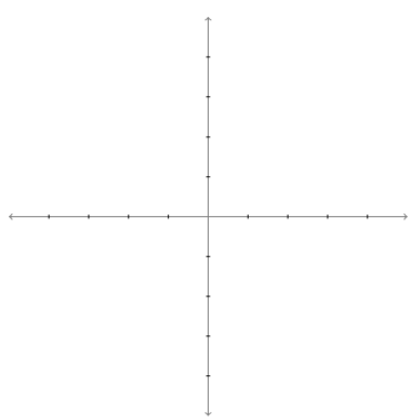
   b. Are there any x values for which $f'(x) = 0$?

   c. This particular function f has an interval on which its derivative $f'(x)$ is constant. What is this interval? What does the derivative function look like there? Estimate the slope of $f(x)$ on that interval.

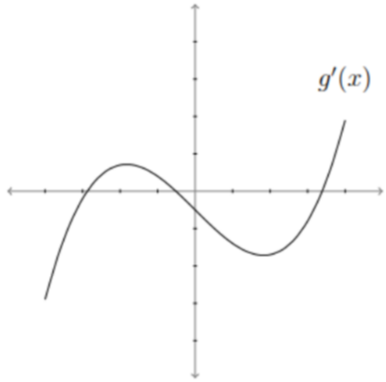
   d. On which interval or intervals is $f'(x)$ positive?

   e. On which interval or intervals is $f'(x)$ negative?

   f. Sketch a graph of the derivative of the function.



2. Below is a graph of a derivative $g'(x)$. Assume this is the entire graph of $g'(x)$. Use the graph to answer the following questions about the original function $g(x)$.



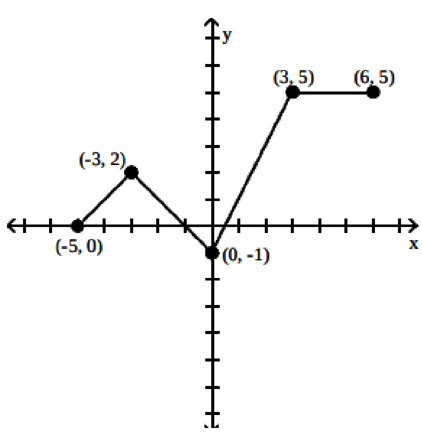
a. On which interval(s) is the original function $g(x)$ increasing?

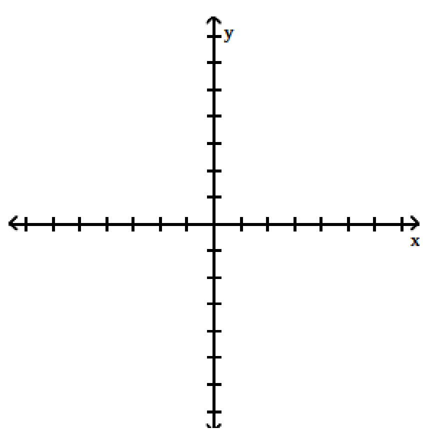
b. On which interval(s) is the original function $g(x)$ decreasing?

c. Now suppose that $g(0) = 0$. Is the function $g(x)$ ever positive? That is, is there any x values such that $g(x) \geq 0$? How do you know?

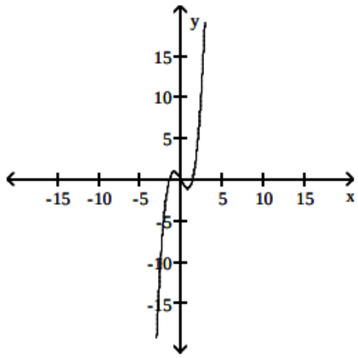
3. Graph the derivative of the function below:

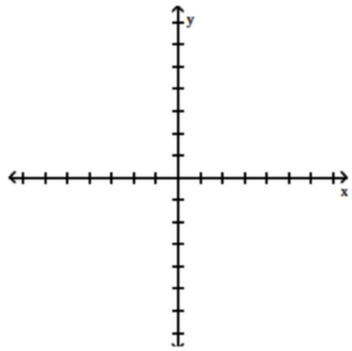
a.



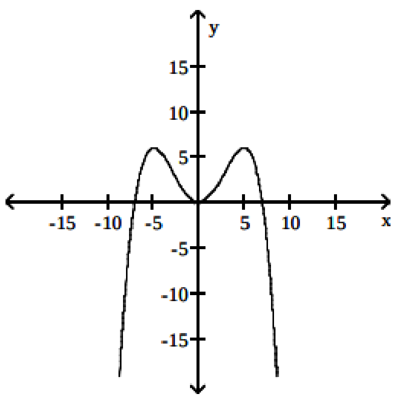


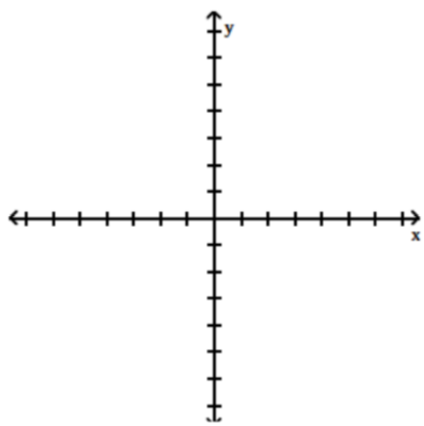
b.





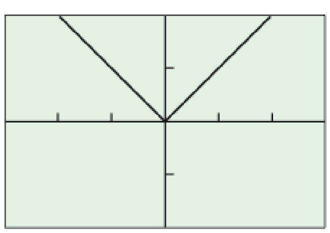
c.



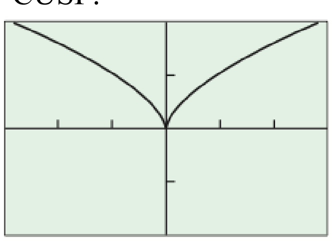


Where $f'(a)$ Does Not Exist

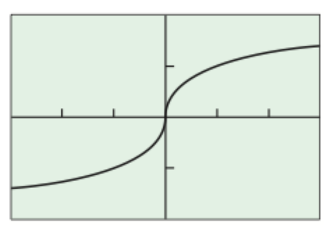
1. CORNER:



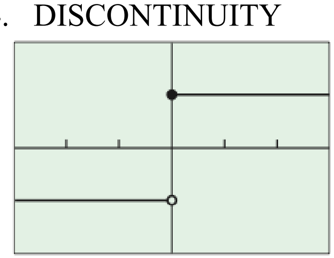
2. CUSP:



3. VERTICAL TANGENT



4. DISCONTINUITY



4. For each function, $f(x)$, determine whether the function is continuous and/or differentiable.

   a. $$f(x) = \begin{cases} x^2, & x \geq 0 \\ x, & x < 0 \end{cases}$$

   b. $$f(x) = \begin{cases} 4 - x^2, & x < 1 \\ 2x + 2, & x \geq 1 \end{cases}$$

5. Find the values of a and b that make the function below differentiable.

$$f(x) = \begin{cases} ax^2 + 10, & x \geq 2 \\ x^2 - 6x + b, & x < 2 \end{cases}$$

THEOREM 1 Differentiability Implies Continuity

If f has a derivative at $x = a$, then f is continuous at $x = a$.

*****Not an If And Only If Statement*******

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Sum Rule: $\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$

Difference Rule: $\frac{d}{dx}[f(x)-g(x)] = f'(x)-g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x)+g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

6.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	-1	1	2
2	1	$\frac{1}{2}$	3	0
3	3	2	1	-2

Given $h(x) = f(x) \cdot g(x)$, find $h'(3)$

7.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-1	2	-1
2	2	-1	1	0
3	1	-1	2	1

Given $h(x) = \frac{f(x)}{g(x)}$, find $h'(3)$

DEFINITION Instantaneous Velocity

The **(instantaneous) velocity** is the derivative of the position function $s = f(t)$ with respect to time. At time t the velocity is

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$

DEFINITION Speed

Speed is the absolute value of velocity.

$$\text{Speed} = |v(t)| = \left|\frac{ds}{dt}\right|$$

DEFINITION Acceleration

Acceleration is the derivative of velocity with respect to time. If a body's velocity at time t is $v(t) = ds/dt$, then the body's acceleration at time t is

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}.$$

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

   (a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

   (b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

   (c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

   (d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.