1. Find the derivative of the following:
   a. $y = 2x^4\sqrt{x^2 - 5}$

BC Topic!

Finding *dy/dx* Parametrically

If all three derivatives exist and $dx/dt \neq 0$,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}. \quad (3)$$

Examples:

1. Find the line tangent to the right-hand hyperbola branch defined parametrically by $x = \sec t, y = \tan t, -\frac{\pi}{2} < t < \frac{\pi}{2}$ at the point $(\sqrt{2},\ 1)$ where $t = \frac{\pi}{4}$

2. Find the equation of the line tangent to the curve defined by $x = \sin 2\pi t, y = \cos 2\pi t$ at the point $t = -\frac{1}{6}$.

1. Find $\frac{dy}{dx}$ of $x^2 - xy + 3y^2 = 7$

2. Given $x^2 + y^2 = 16$, find the equation of the tangent line at $(3, \sqrt{7})$.

Challenge Problems!

1. Find $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 8$

Theorem 1 **Derivatives of Inverse Functions**	Derivation:
Let $g(x)$ be the inverse of $f(x)$. $g'(x) = \frac{1}{f'(g(x))}$	

1.

Let f and g be **inverse** functions.

The following table lists a few values of f, g, and f'.

x	$f(x)$	$g(x)$	$f'(x)$
-2	2	5	1
5	-2	-4	$-\frac{1}{2}$

$g'(-2) =$ ____

2.

Let f and g be **inverse** functions.

The following table lists a few values of f, g, and g'.

x	$f(x)$	$g(x)$	$g'(x)$
-3	5	4	$-\frac{1}{4}$
4	-3	2	2

$f'(4) =$ ____

3.

Let $h(x) = 7 - x - 2x^5$ and let f be the inverse function of h. Notice that $h(-1) = 10$.

$f'(10) =$ ____

4.

Let $g(x) = x^5 + 3x$ and let h be the inverse function of g. Notice that $g(1) = 4$.

$h'(4) =$ ____

Derivative Rules:

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(a^x) = a^x \cdot \ln a$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\ln x) = \frac{1}{x}, \quad x > 0$$

$$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\cdot\sqrt{x^2-1}} \qquad \frac{d}{dx}(\ln|x|) = \frac{1}{x}, \quad x \neq 0$$

$$\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{|x|\cdot\sqrt{x^2-1}} \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x\cdot\ln a}, \quad x > 0$$

$$\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

Examples:

1. Find the derivative of y with respect to the appropriate variable.

   a. $y = \sin^{-1}\sqrt{2t}$

   b. $y = x\sin^{-1}x + \sqrt{1-x^2}$

1. Find the derivative of the following equations:

   a. $y = \ln(x^3 + 3x - 2)$

   c. $y = 3^{x+2}$

   b. $y = e^{3x^2}$

   d. $y = x^{\sin x}$