AB Calculus
Chapter 4 Review

1. List two limit definition of derivative:

2. What are some other words for derivative

3. What is the Product Rule and the Quotient Rule

4. Find the derivative of the following functions:

a. x^a

b. $y = \sin x$

c. $y = \cos x$

d. $y = \tan x$

e. $y = \csc x$

f. $y = \sec x$

g. $y = \cot x$

h. $y = \sqrt{x}$

i. $y = \frac{1}{\sqrt{x}}$

j. $y = c$

k. $y = e^x$

l. $y = a^x$

m. $y = \ln x$

n. $y = \log_a x$

o. $y = \arcsin x$

p. $y = \arccos x$

q. $y = \arctan x$

r. $y = \csc^{-1} x$

s. $y = \sec^{-1} x$

t. $y = \cot^{-1} x$

u. If f and g are inverse functions what is $g'(x)$ =?

5.

1988 AB 24

$\frac{d}{dx}\left(x^{\ln x}\right) =$

(A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)\left(x^{\ln x}\right)$ (D) $(\ln x)\left(x^{\ln x - 1}\right)$ (E) $2(\ln x)\left(x^{\ln x}\right)$

6. Find $\frac{d^2y}{dx^2}$ where $y^2 - xy = 8$

6. Use the table below to find the specified derivatives.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	$\frac{1}{3}$	-2	-3
2	3	$\frac{1}{2}$	4	0
3	1	-2	5	-1

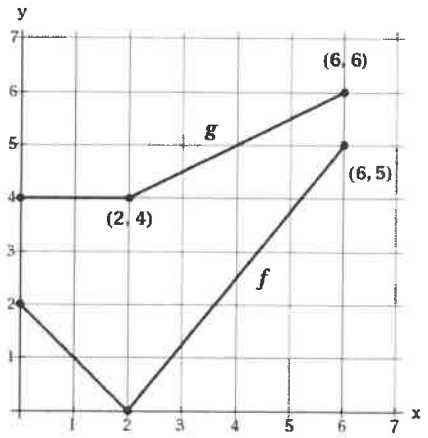
a. If $h(x) = g(f(x))$, find $h'(3)$

b. If $h(x)$ is the inverse of $f(x)$, find $h'(1)$

7. Let $r(x) = f(g(x))$ and $s(x) = g(f(x))$ where f and g are shown in the figure below.

a. Find $r'(1)$

b. Find $s'(4)$



8) If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$ 2003 AB 1

(A) $(3x^2)^2$ (B) $2(x^3 + 1)$ (C) $2(3x^2 + 1)$ (D) $3x^2(x^3 + 1)$ (E) $6x^2(x^3 + 1)$

9) If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$ 2003 AB 14

(A) $2x \cos 2x$

(B) $4x \cos 2x$

(C) $2x(\sin 2x + \cos 2x)$

(D) $2x(\sin 2x - x \cos 2x)$

(E) $2x(\sin 2x + x \cos 2x)$

10) $\frac{d}{dx}\cos^2(x^3) =$ 1997 AB 7

(A) $6x^2\sin(x^3)\cos(x^3)$

(B) $6x^2\cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2\sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

11) If $f(x)=(x-1)(x^2+2)^3$, then $f'(x)=$ 2008 AB 3

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

(D) $(x^2+2)^2(7x^2-6x+2)$

(E) $-3(x-1)(x^2+2)^2$

12) If $f(x)=(x^2-2x-1)^{\frac{2}{3}}$, then $f'(0)$ is 1993 AB 24 Calc

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

13) If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

1997 AB 2

(A) $\dfrac{3x-3}{\sqrt{2x-3}}$

(B) $\dfrac{x}{\sqrt{2x-3}}$

(C) $\dfrac{1}{\sqrt{2x-3}}$

(D) $\dfrac{-x+3}{\sqrt{2x-3}}$

(E) $\dfrac{5x-6}{2\sqrt{2x-3}}$

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challenge ↓

2003 AB 89 Calc

41) Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = 3x$

(B) $y - 3 = -5(x - 2)$

(C) $y - 6 = -5(x - 2)$

(D) $y - 6 = -7(x - 2)$

(E) $y - 6 = -10(x - 2)$

15) If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$ 1998 AB 6

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

2003 AB 26

16) What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

17) If $3x^2+2xy+y^2=2$, then the value of $\frac{dy}{dx}$ at $x=1$ is 1969 AB 5

(A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

18) If $x^2+xy+y^3=0$, then, in terms of x and y, $\frac{dy}{dx}=$ 1985 AB 13

(A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

19) Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

2003 AB 27

20) An equation for a tangent to the graph of $y = \arcsin\frac{x}{2}$ at the origin is

1969 AB 20

(A) $x-2y=0$ (B) $x-y=0$ (C) $x=0$ (D) $y=0$ (E) $\pi x-2y=0$

21) If $y=\arctan(\cos x)$, then $\frac{dy}{dx}=$

1985 AB 26

(A) $\frac{-\sin x}{1+\cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2+1}$ (E) $\frac{1}{1+\cos^2 x}$

22) If $y = e^{nx}$, then $\frac{d^n y}{dx^n} =$

1973 AB 36

(A) $n^n e^{nx}$ (B) $n!e^{nx}$ (C) ne^{nx} (D) $n^n e^x$ (E) $n!e^x$

2008 AB 82 calc

23) A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 3$?

(A) –0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

1998 AB 16

24) If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$

25) Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

1997 AB 80 calc

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

26) Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

1998 AB 77 Calc

(A) –0.701
(B) –0.567
(C) –0.391
(D) –0.302
(E) –0.258

27) If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

★ try w/o calc ★
1993 AB 31 Calc

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

28) If $f(x)=x^2+2x$, then $\frac{d}{dx}\left(f(\ln x)\right)=$ 2008 AB 13

(A) $\frac{2\ln x+2}{x}$ (B) $2x\ln x+2$ (C) $2\ln x+2$ (D) $2\ln x+\frac{2}{x}$ (E) $\frac{2x+2}{x}$

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29) The slope of the line <u>normal</u> to the graph of $y=2\ln(\sec x)$ at $x=\frac{\pi}{4}$ is 1993 AB 16 Calc

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

30) The $\lim_{h\to 0}\dfrac{\tan 3(x+h)-\tan 3x}{h}$ is 1988 AB 29

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

31) The slope of the line tangent to the graph of $y=\ln\left(x^2\right)$ at $x=e^2$ is 1973 AB 3

(A) $\dfrac{1}{e^2}$ (B) $\dfrac{2}{e^2}$ (C) $\dfrac{4}{e^2}$ (D) $\dfrac{1}{e^4}$ (E) $\dfrac{4}{e^4}$

32) If $f(x)=\tan(2x)$, then $f'\left(\dfrac{\pi}{6}\right)=$ 1998 AB 28

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

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33)

5. Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.