

AB Calculus
Chapter 4 Review

1. List two limit definition of derivative:

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

2. What are some other words for derivative

slope, instantaneous rate of change

3. What is the Product Rule and the Quotient Rule

$$(uv)' = u'v + uv'$$

$$\frac{u'v + uv'}{\sqrt{u^2}}$$

4. Find the derivative of the following functions:

a. x^a

$$nx^{a-1}$$

h. $y = \sqrt{x}$ $y' = \frac{1}{2}x^{-\frac{1}{2}}$

b. $y = \sin x$

$$y' = \cos x$$

i. $y = \frac{1}{\sqrt{x}}$ $y' = -\frac{1}{2}x^{-\frac{3}{2}}$

c. $y = \cos x$

$$y' = -\sin x$$

j. $y = c$ $y' = 0$

d. $y = \tan x$

$$y' = \sec^2 x$$

k. $y = e^x$ $y' = e^x$

e. $y = \csc x$

$$y' = -\csc x \cot x$$

l. $y = a^x$ $y' = a^x \ln a$

f. $y = \sec x$

$$y' = \sec x \tan x$$

m. $y = \ln x$

$$y' = \frac{1}{x}$$

g. $y = \cot x$

$$y' = -\csc^2 x$$

n. $y = \log_a x$

$$y' = \frac{1}{x \ln a}$$

*chain rule

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o. $y = \arcsin x$

$$\frac{1}{\sqrt{1-x^2}}$$

p. $y = \arccos x$

$$-\frac{1}{\sqrt{1-x^2}}$$

q. $y = \arctan x$

$$\frac{1}{1+x^2}$$

r. $y = \csc^{-1} x$

s. $y = \sec^{-1} x$

t. $y = \cot^{-1} x$

$$-\frac{1}{1+x^2}$$

u. If f and g are inverse

$$g'(x) = \frac{1}{f'(g(x))}$$

functions what is $g'(x) = ?$

5.

1988 AB 24

$$\frac{d}{dx}(x^{\ln x}) =$$

- (A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)(x^{\ln x})$ (D) $(\ln x)(x^{\ln x-1})$ (E) $2(\ln x)(x^{\ln x})$

$$\begin{aligned} y &= x^{\ln x} \\ \ln y &= \ln x (\ln x) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{\ln x}{x} + \frac{1}{x} x^{\ln x} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= x^{\ln x} \left(\frac{\ln x}{x} + \frac{1}{x} x^{\ln x} \right) \\ &= \frac{2}{x} (\ln x) x^{(\ln x)} \end{aligned}$$

6. Find $\frac{d^2y}{dx^2}$ where $y^2 - xy = 8$

$$2y \frac{dy}{dx} - \left(y - x \frac{dy}{dx} \right) = 0$$

$$\frac{dy}{dx} = \frac{y}{2y-x}$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{(2y-x)(\frac{dy}{dx}) - y(2\frac{dy}{dx} - 1)}{(2y-x)^2} \\ &= \frac{2y - \left(\frac{dy}{dx} \right) - 2y \left(\frac{3}{2y-x} \right) + y}{(2y-x)^2} \end{aligned}$$

$$= y - \frac{2y^2}{2y-x} + y$$
$$-\frac{(2y-x)^2}{(2y-x)^2}$$

$$= \frac{2y - \frac{2y^2}{2y-x}}{(2y-x)^2}$$

$$= \frac{2y(2y-x) - 2y^2}{2y-x} \cdot \frac{1}{(2y-x)^2}$$

$$= \frac{4y^2 - 2yx - 2y^2}{(2y-x)^3}$$

$$= \frac{2y^2 - 2yx}{(2y-x)^3}$$

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6. Use the table below to find the specified derivatives.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	$\frac{1}{3}$	-2	-3
2	3	$\frac{1}{2}$	4	0
3	1	-2	5	-1

$$f(3, 1) \\ h(1, 3)$$

- a. If $h(x) = g(f(x))$, find $h'(3)$

$$\begin{aligned} h'(3) &= g'(f(3)) f'(3) \\ &= g'(1)(-2) \\ &= (-3)(-2) \\ &= 6 \end{aligned}$$

- b. If $h(x)$ is the inverse of $f(x)$, find $h'(1)$

$$\begin{aligned} h'(1) &= \frac{1}{f'(h(1))} \\ &= \frac{1}{f'(3)} \\ &= -\frac{1}{-2} = -\frac{1}{2} \end{aligned}$$

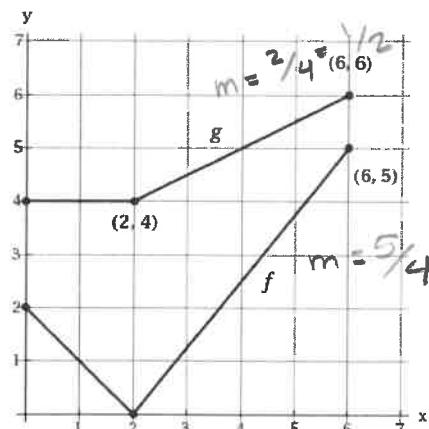
7. Let $r(x) = f(g(x))$ and $s(x) = g(f(x))$ where f and g are shown in the figure below.

- a. Find $r'(1)$

$$\begin{aligned} r'(x) &= f'(g(x)) g'(x) \\ r'(1) &= f'(g(1)) g'(1) \\ &= f'(4)(0) \\ &= 0 \end{aligned}$$

- b. Find $s'(4)$

$$\begin{aligned} s'(4) &= g'(f(4)) f'(4) \\ &= g'(2.5)(5/4) \\ &= \frac{1}{2} \cdot \frac{5}{4} \\ &= \frac{5}{8} \end{aligned}$$



8) If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$ 2003 AB 1

- (A) $(3x^2)^2$ (B) $2(x^3 + 1)$ (C) $2(3x^2 + 1)$ (D) $3x^2(x^3 + 1)$ (E) $6x^2(x^3 + 1)$

$$y' = 2(x^3 + 1)(3x^2)$$

$$= 6x^2(x^3 + 1)$$

9) If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$ 2003 AB 14

- (A) $2x \cos 2x$
(B) $4x \cos 2x$
(C) $2x(\sin 2x + \cos 2x)$
(D) $2x(\sin 2x - x \cos 2x)$
(E) $2x(\sin 2x + x \cos 2x)$

$$y' = 2x \sin 2x + 2x^2 \cos 2x$$

$$= 2x(\sin 2x + x^2 \cos 2x)$$

(10) $\frac{d}{dx} \cos^2(x^3) =$ 1997 AB 7

- (A) $6x^2 \sin(x^3) \cos(x^3)$
(B) $6x^2 \cos(x^3)$
(C) $\sin^2(x^3)$
(D) $-6x^2 \sin(x^3) \cos(x^3)$
(E) $-2 \sin(x^3) \cos(x^3)$

$$-6x^2 \cos(x^3) \sin(x^3)$$

11) If $f(x) = (x-1)(x^2+2)^3$, then $f'(x) = \text{_____}$ 2008 AB 3

- (A) $6x(x^2 + 2)^2$
(B) $6x(x - 1)(x^2 + 2)^2$
(C) $(x^2 + 2)^2(x^2 + 3x - 1)$
(D) $(x^2 + 2)^2(7x^2 - 6x + 1)$
(E) $-3(x - 1)(x^2 + 2)^2$

$$\begin{aligned}
 f'(x) &= (1)(x^2 + 2)^3 + (x - 1) 3(x^2 + 2)^2 (2x) \\
 &= (x^2 + 2)^3 + 6x(x - 1)(x^2 + 2)^2 \\
 &= (x^2 + 2)^2 ((x^2 + 2) + 6x^2 - 6x) \\
 &= (x^2 + 2)^2 (7x^2 - 6x + 2)
 \end{aligned}$$

(2) If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then $f'(0)$ is 1993 AB 24 Calc

- (A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

$$f'(x) = \frac{2}{3}(x^2 - 2x - 1)^{-\frac{1}{3}}(2x - 2)$$

$$f'(0) = \frac{2(z(0)-2)}{3(z^2-z(0)-1)^{1/3}} = \frac{-4}{3(-1)^{1/3}} = \frac{-4}{-3} = \frac{4}{3}$$

- 13) If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

1997 AB 2

(A) $\frac{3x-3}{\sqrt{2x-3}}$

(B) $\frac{x}{\sqrt{2x-3}}$

(C) $\frac{1}{\sqrt{2x-3}}$

(D) $\frac{-x+3}{\sqrt{2x-3}}$

(E) $\frac{5x-6}{2\sqrt{2x-3}}$

$$f'(x) = (1)\sqrt{2x-3} + \frac{1}{2}x(2x-3)^{-1/2}(2)$$

$$= \sqrt{2x-3} + \frac{x}{\sqrt{2x-3}}$$

$$= \frac{2x-3+x}{\sqrt{2x-3}}$$

$$= \frac{3x-3}{\sqrt{2x-3}}$$

challenge ↴

2003 AB 89 Calc

Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = 2$?

- (A) $y = 3x$
 (B) $y - 3 = -5(x - 2)$
 (C) $y - 6 = -5(x - 2)$
 (D) $y - 6 = -7(x - 2)$
 (E) $y - 6 = -10(x - 2)$

$$g'(x) = (1)f(x) + xf'(x)$$

$$g'(2) = f(2) + 2f'(2)$$

$$= 3 + 2(-5)$$

$$= 3 - 10$$

$$= -7$$

$$g(2) = 2f(2)$$

$$= 6$$

$$(2, 6)$$

$$y - 6 = -7(x - 2)$$

If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

1998 AB 6

- (A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

$$2x + y + x \frac{dy}{dx} = 0$$

$$2^2 + 2y = 10$$

$$2(2) + 3 + 2 \frac{dy}{dx} = 0$$

$$2y = 6$$

$$y = 3$$

$$\frac{dy}{dx} = -\frac{7}{2}$$

2003 AB 26

16) What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$?

- (A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

$$6y \frac{dy}{dx} - 4x = -2(y + x \frac{dy}{dx})$$

$$6(2) \frac{dy}{dx} - 4(3) = -2(2 + 3 \frac{dy}{dx})$$

$$12 \frac{dy}{dx} - 12 = -4 - 6 \frac{dy}{dx}$$

$$18 \frac{dy}{dx} = 8$$

$$\frac{dy}{dx} = \frac{8}{18} = \frac{4}{9}$$

7) If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x=1$ is

1969 AB 5

- (A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

$$6x + 2(y + x \frac{dy}{dx}) + 2y \frac{dy}{dx} = 0$$

$$3(1)^2 + 2y + y^2 = 2$$

$$6(1) + 2(-1) + 2(1) \frac{dy}{dx} + 2(-1) \frac{dy}{dx} = 0$$

$$y^2 + 2y + 1 = 0$$

$$6 - 2 + 2 \frac{dy}{dx} - 2 \frac{dy}{dx} = 0$$

$$(y+1)^2 = 0$$

can check

$$\left. \frac{dy}{dx} \right|_{(1, -1)} = \frac{-6x - 2y}{2x + 2y} = \frac{-3x - y}{x + y} = \frac{-3 + 1}{1 - 1} = \frac{-2}{0}$$

18) If $x^2 + xy + y^3 = 0$, then, in terms of x and y , $\frac{dy}{dx} =$

1985 AB 13

- (A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

$$2x + x \frac{dy}{dx} + y + 3y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 3y^2}$$

- 19) Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?
 (A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

2003 AB 27

$$g'(2) = \frac{1}{f'(g(2))}$$

$$= \frac{1}{f'(1)} = \frac{1}{4}$$

$$f'(x) = 3x^2 + 1$$

$$f'(1) = 4$$

- 20) An equation for a tangent to the graph of $y = \arcsin \frac{x}{2}$ at the origin is

1969 AB 20

- (A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$ (D) $y = 0$ (E) $\pi x - 2y = 0$

$$y' = \frac{1}{\sqrt{1 - (\frac{x}{2})^2}} \cdot \frac{1}{2}$$

$$y'(0) = \frac{1}{2\sqrt{1-0}} = \frac{1}{2}$$

$$y - 0 = \frac{1}{2}(x - 0)$$

$$-\frac{1}{2}x + y = 0$$

$$x - 2y = 0$$

- 21) If $y = \arctan(\cos x)$, then $\frac{dy}{dx} =$

1985 AB 26

- (A) $\frac{-\sin x}{1 + \cos^2 x}$ (B) $-(\text{arcsec}(\cos x))^2 \sin x$ (C) $(\text{arcsec}(\cos x))^2$
 (D) $\frac{1}{(\text{arccos } x)^2 + 1}$ (E) $\frac{1}{1 + \cos^2 x}$

$$y' = \frac{1}{1 + \cos^2 x} (-\sin x)$$

$$= \frac{-\sin x}{1 + \cos^2 x}$$

22) If $y = e^{nx}$, then $\frac{d^n y}{dx^n} =$

1973 AB 36

(A) $n^n e^{nx}$

(B) $n!e^{nx}$

(C) ne^{nx}

(D) $n^n e^x$

(E) $n!e^x$

$$y' = n e^{nx}$$

$$y'' = n^2 e^{nx}$$

$$y''' = n^3 e^{nx}$$

2008 AB 32

23) A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 3$? calc

- (A) -0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

$$a(t) = - (1.01)^{-t^2} \ln(1.01) (-2t)$$

$$a(3) = 2(3) (1.01)^{-3^2} \ln(1.01)$$

* can use
calc to get
derivative

24) If $f(x) = \sin(e^{-x})$, then $f'(x) =$

1998 AB 16

$$f'(x) = \cos(e^{-x})(-e^{-x})$$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x} \cos(e^{-x})$

(E) $-e^{-x} \cos(e^{-x})$

5)

Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

1997 AB
80
calc

- (A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

$$f'(x) = 2e^{4x^2} (8x)$$

$$3 = 16x e^{4x^2}$$

* can use calc to get

derivative

on calc:

OR

$$\text{solve } \left(3 = \frac{d}{dx} (2e^{4x^2}), x \right)$$

26)

Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

1998 AB
77
calc

- (A) -0.701
(B) -0.567
(C) -0.391
(D) -0.302
(E) -0.258

$$f'(x) = 6e^{2x}$$

$$g'(x) = 18x^2$$

$$6e^{2x} = 18x^2$$

$$e^{2x} = 3x^2$$

OR

$$\text{solve } \left(\frac{d}{dx} (3e^{2x}) = \frac{d}{dx} (6x^3), x \right)$$

*try w/o calc *

27) If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

1993 AB 31
calc

- (A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2} e^{3\ln(x^2)}$ (C) $6(\ln x) e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

$$f(x) = e^{\ln x^6}$$

$$= x^6$$

$$f'(x) = 6x^5$$

- 28) If $f(x) = x^2 + 2x$, then $\frac{d}{dx}(f(\ln x)) =$

2008 AB 13

- (A) $\frac{2 \ln x + 2}{x}$ (B) $2x \ln x + 2$ (C) $2 \ln x + 2$ (D) $2 \ln x + \frac{2}{x}$ (E) $\frac{2x + 2}{x}$

$$f(\ln x) = (\ln x)^2 + 2 \ln x$$

$$f'(\ln x) = \frac{2(\ln x)}{x} + \frac{2}{x} = \frac{2 \ln x + 2}{x}$$

- 29) The slope of the line normal to the graph of $y = 2 \ln(\sec x)$ at $x = \frac{\pi}{4}$ is

1993 AB 16
calc

- (A) -2
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 2
(E) nonexistent

$$y' = \frac{2}{\sec x} (\sec x \tan x)$$

$$= 2 \tan x$$

$$y'(\frac{\pi}{4}) = 2 \tan \frac{\pi}{4}$$

$$= 2(1)$$

$$= 2$$

$$m_{\text{normal}} = -\frac{1}{2}$$

The $\lim_{h \rightarrow 0} \frac{\tan 3(x+h) - \tan 3x}{h}$ is

1988 AB 29

- (A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

$$f(x) = \tan 3x$$

$$f'(x) = 3\sec^2 3x$$

31) The slope of the line tangent to the graph of $y = \ln(x^2)$ at $x = e^2$ is

1973 AB 3

- (A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

$$y' = \frac{2x}{x^2}$$

$$y'(e^2) = \frac{2e^2}{e^4} = \frac{2}{e^2}$$

32) If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

1998 AB 28

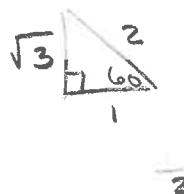
- (A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

$$f'(x) = 2\sec^2 2x$$

$$f'\left(\frac{\pi}{6}\right) = 2\left(\sec \frac{\pi}{3}\right)^2$$

$$= 2(2)^2$$

$$= 8$$



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2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

33)

5. Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y-x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

2 points

1: $0 = \frac{y}{2y-x}$

1: explanation

(a) $y^2 = 2 + xy$

2 points

(c) $0 = \frac{y}{2y-x}$

$2y \frac{dy}{dx} = y + x \frac{dy}{dx}$

1: implicit
Diff

$0 = y$

$2y \frac{dy}{dx} - x \frac{dy}{dx} = y$

1: solves for
 $\frac{dy}{dx}$

The curve has no

$\frac{dy}{dx} (2y-x) = y$

horizontal tangents b/c

$\frac{dy}{dx} = \frac{y}{2y-x}$

$0^2 \neq 2 + 0x$ for any x

$3^2 = 2 + 3x$

(d) $2y \frac{dy}{dt} = \frac{dx}{dt} y + x \frac{dy}{dt}$

$\frac{7}{3} = x$

$2(3)(6) = \frac{dx}{dt} (3) + x (6)$

(b)

$\frac{1}{2} = \frac{y}{2y-x}$

2 points

$3(6) = 3 \frac{dx}{dt} + \frac{7}{3}(6)$

$2y-x = 2y$

1: $\frac{y}{2y-x} = \frac{1}{2}$

$22 = 3 \frac{dx}{dt}$

$x = 0$

1: answer

$\frac{22}{3} = \frac{dx}{dt}$

$y^2 = 2 + 0(y)$

$y = \pm\sqrt{2}$

3 points

$(0, \sqrt{2})$ and $(0, -\sqrt{2})$

1: solved for x

$\left. \frac{dx}{dt} \right|_{t=5} = \frac{22}{3}$

1: chain rule

1: answer