Calculator Active if labeled as "Calc"
★ can find derivative on calculator ★

1) A point moves on the x-axis in such a way that its velocity at time t $(t>0)$ is given by $v=\frac{\ln t}{t}$. At what value of t does v attain its maximum?

1969 BC 19

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

$$v' = \frac{t(1/t) - \ln t(1)}{t^2}$$

$$0 = 1 - \ln t$$

$$\ln t = 1 \qquad t = e$$

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2) At what values of x does $f(x)=3x^5-5x^3+15$ have a relative maximum?

1985 BC 2

(A) -1 only (B) 0 only (C) 1 only (D) -1 and 1 only (E) -1, 0 and 1

$$f'(x) = 15x^4 - 15x^2$$

$$0 = 15x^2(x^2-1)$$

$$x = 0, \pm 1$$

$+ \quad - \quad - \quad +$ f'

$-1 \quad 0 \quad 1$

$f(0) = 15$

$f(1) = 3 - 5 + 15 = 13$

$f(-1) = -3 + 5 + 15 = 17$

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3) For what value of k will $x+\frac{k}{x}$ have a relative maximum at $x=-2$?

1969 BC 7

(A) -4 (B) -2 (C) 2 (D) 4 (E) None of these

$$f'(x) = 1 - \frac{k}{x^2}$$

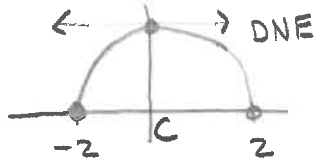
$$0 = 1 - k/(-2)^2$$

$$1 = k/4$$

$$4 = k$$

4) The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

2008 BC 89 Calc



(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

*Rolles Thm broken

foundation for MVT

1973 BC 18

5) Let g be a continuous function on the closed interval $[0,1]$. Let $g(0) = 1$ and $g(1) = 0$. Which of the following is NOT necessarily true?

(A) There exists a number h in $[0,1]$ such that $g(h) \ge g(x)$ for all x in $[0,1]$. True by EVT

(B) For all a and b in $[0,1]$, if $a = b$, then $g(a) = g(b)$. ✓ True b/c g is a function

(C) There exists a number h in $[0,1]$ such that $g(h) = \frac{1}{2}$. ✓ True b/c IVT

(D) There exists a number h in $[0,1]$ such that $g(h) = \frac{3}{2}$.

(E) For all h in the open interval $(0,1)$, $\lim_{x \to h} g(x) = g(h)$. ✓ True b/c g continuous

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6) The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between $(0,0)$ and $(4,2)$. What are the coordinates of this point? 1969 BC 3

(A) $(2,1)$

(B) $(1,1)$

(C) $(2,\sqrt{2})$

(D) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

(E) None of the above

$f'(c) = \frac{f(4) - f(0)}{4 - 0} = \frac{2-0}{4} = \frac{1}{2}$

$y' = \frac{1}{2} x^{-1/2}$

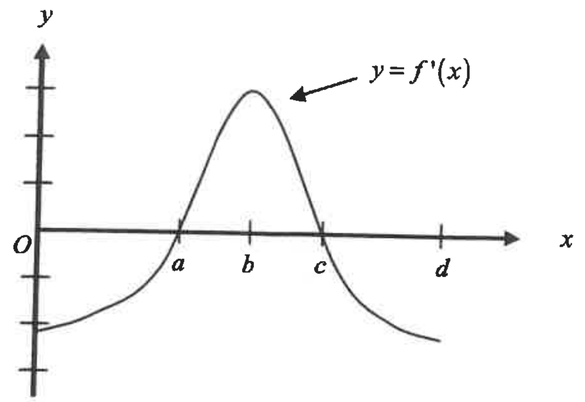
$\frac{1}{2} = \frac{1}{2} c^{-1/2}$

$1 = c^{-1/2}$

$1 = c^{1/2}$

$1 = c$

$y(1) = \sqrt{1} = 1$



2008 BC 76 Calc

7) The graph of f', the derivative of a function f, is shown above. The domain of f is the open interval $0<x<d$. Which of the following statements is true?

(A) f has a local minimum at $x=c$. pos to neg ↗ ↘

(B) f has a local maximum at $x=b$. no greatest pos slope at $x=b$

(C) The graph of f has a point of inflection at $(a, f(a))$. no $f''=0$ and changes sign = inflection

(D) The graph of f has a point of inflection at $(b, f(b))$. yes $f''=0$ and changes sign

(E) The graph of f is concave up on the open interval (c, d). $f'<0$ so concave down

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8) Which of the following is true about the graph of $y=\ln\left|x^2-1\right|$ in the interval $(-1,1)$? 1973 BC 26

(A) It is increasing. no
(B) It attains a relative ~~minimum~~ max at $(0,0)$.
(C) It has a range of all real numbers. no taking ln(neg)
(D) It is concave down.
(E) It has an asymptote of $x=0$. no $x=1$

$$y'=\frac{2x}{x^2-1}$$

$$0=\frac{2x}{x^2-1}$$

↘ ↗ ↘ ↗
− + − + f'
−1 0 1

c.p. at $x=0$
$x=\pm1$

$$y''=\frac{(x^2-1)(2)-(2x)(2x)}{(x^2-1)^2}=\frac{2x^2-2-4x^2}{(x^2-1)^2}=\frac{-2x^2-2}{(x^2-1)^2}$$

$$=\frac{\overbrace{-2(x^2+1)}^{\text{pos}}}{\underbrace{(x^2-1)^2}_{\text{pos}}}$$

*always neg

9) If $f(x)=x^2e^x$, then the graph of f is decreasing for all x such that

1993 BC 22
Calc

(A) $x<-2$ (B) $-2<x<0$ (C) $x>-2$ (D) $x<0$ (E) $x>0$

$f'(x) = 2xe^x + x^2e^x$

$0 = e^x x(2+x)$

$x = 0, -2$

+ − + f'

−2 0

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10) If $f(x)=x+\frac{1}{x}$, then the set of values for which f increases is

1973 BC 3

(A) $(-\infty,-1]\cup[1,\infty)$ (B) $[-1,1]$ (C) $(-\infty,\infty)$

(D) $(0,\infty)$ (E) $(-\infty,0)\cup(0,\infty)$

$f'(x) = 1 - x^{-2}$

$0 = 1 - x^{-2}$

$1 = x^{-2}$

$1 = x^2$

$\pm 1 = x$

c.p. $x = 0, \pm 1$

$f'(x) = 1 - \frac{1}{x^2}$

+ − − + f'

−1 0 1

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11) Let f be the function given by $f(x)=\cos(2x)+\ln(3x)$. What is the least value of x at which the graph of f changes concavity?

1997 BC 80
Calc

(A) 0.56 (B) 0.93 (C) 1.18 (D) 2.38 (E) 2.44

$f'(x) = \frac{1}{x} - 2\sin 2x$

$f''(x) = -4\cos 2x - \frac{1}{x^2}$

$0 = f''(x)$

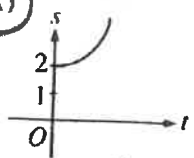
$x = 93.463,\ 3.935,\ 2.333,\ 0.932$
$-0.932,\ -3.935,\ -2.333,\ -93.463$

* too many p.o.i candidates
$\Rightarrow$ graph f''

12) A particle starts from rest at the point $(2,0)$ and moves along the x-axis with a constant positive acceleration for time $t \geq 0$. Which of the following could be the graph of the distance $s(t)$ of the particle from the origin as a function of time t?

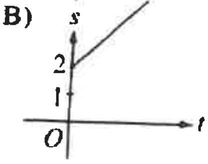
1998 BC 90 Calc

(A)

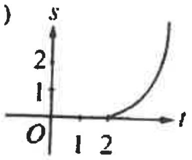


* second derivative pos
↳ concave up

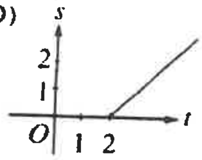
(B)



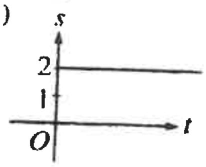
(C)



(D)



E)



13) If f is the function defined by $f(x)=3x^5-5x^4$, what are all the x-coordinates of points of inflection for the graph of f?

1998 BC
16

(A) -1　　(B) 0　　(C) 1　　(D) 0 and 1　　(E) -1, 0, and 1

$f'(x) = 15x^4 - 20x^3$

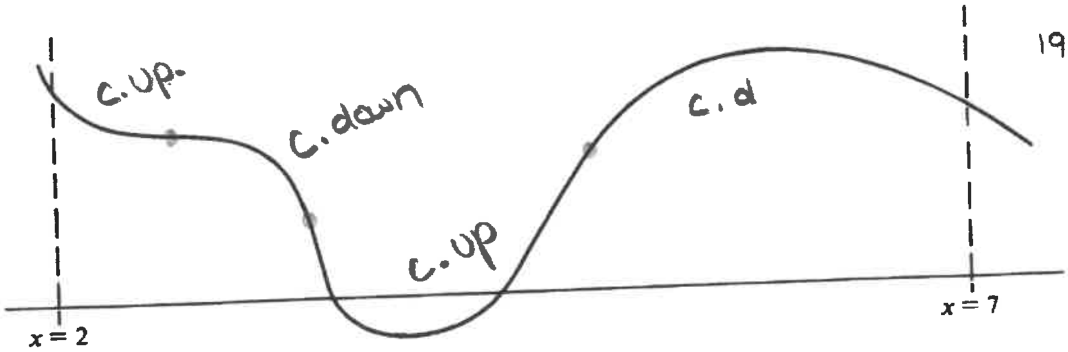
$f''(x) = 60x^3 - 60x^2$

$0 = 60x^2(x-1)$

$x = 0, 1$

$$\begin{array}{ccccc} - & & - & & + \\ \hline & 0 & & 1 & \end{array} \quad f''$$

1985 BC 20



4) The graph of $y = f(x)$ on the closed interval $[2,7]$ is shown above. How many points of inflection does this graph have on this interval?

(A) One (B) Two (C) Three (D) Four (E) Five

15) Let f be the function with derivative defined by $f'(x) = \sin(x^3)$ on the interval $-1.8 < x < 1.8$. How many points of inflection does the graph of f have on this interval?

2003 BC Calc

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

$f''(x) = 3x^2 \cos x^3$

$0 = 3x^2 \cos x^3$

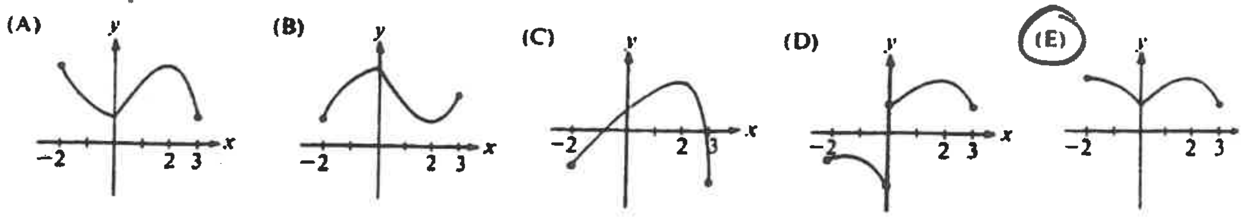
graph on calc

see 5 x values where $f'' = 0$

but 4 x values where $f'' = 0$ and changes sign

Let f be a function that is continuous on the closed interval $[-2,3]$ such that $f'(0)$ does not exist, $f'(2)=0$, and $f''(x)<0$ for all x except $x=0$. Which of the following could be the graph of f? 1985 BC 43

cp c.d



must be continuous

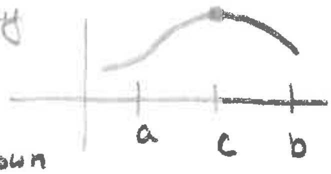
17) Let f be a function defined and continuous on the closed interval $[a,b]$. If f has a relative maximum at c and $a<c<b$, which of the following statements must be true? 1998 BC 23

not necessarily b/c doesn't say differentiable

I. $f'(c)$ exists.

II. If $f'(c)$ exists, then $f'(c)=0$.

III. If $f''(c)$ exists, then $f''(c)\le 0$. concave down



(A) II only (B) III only (C) I and II only (D) I and III only (E) II and III only

18) Let f be a continuous function on the closed interval $[-3,6]$. If $f(-3)=-1$ and $f(6)=3$, then the Intermediate Value Theorem guarantees that

1997 BC 81

Calc

(A) $f(0)=0$

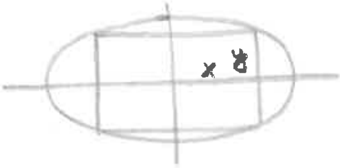
(B) $f'(c)=\frac{4}{9}$ for at least one c between -3 and 6

Chapter 3

(C) $-1\le f(x)\le 3$ for all x between -3 and 6

(D) $f(c)=1$ for at least one c between -3 and 6

(E) $f(c)=0$ for at least one c between -1 and 3



19) What is the area of the largest rectangle that can be inscribed in the ellipse $4x^2 + 9y^2 = 36$? 1988 BC 45

(A) $6\sqrt{2}$ (B) 12 (C) 24 (D) $24\sqrt{2}$ (E) 36

see next page

20) Consider all right circular cylinders for which the sum of the height and circumference is 30 centimeters. What is the radius of the one with maximum volume? 1993 BC 36 Calc

(A) 3 cm (B) 10 cm (C) 20 cm (D) $\frac{30}{\pi^2}$ cm (E) $\frac{10}{\pi}$ cm

$h + c = 30\text{cm}$

$h + 2\pi r = 30$

$h = 30 - 2\pi r$

$V = \pi r^2 h$

$V = \pi r^2(30 - 2\pi r)$

$= 30\pi r^2 - 2\pi^2 r^3$

$V' = 60\pi r - 6\pi^2 r^2$

$0 = 6\pi r(10 - \pi r)$

$r = 0, \ 10/\pi$

$$4x^2 + 9y^2 = 36$$

$$9y^2 = 36 - 4x^2$$

$$y = \sqrt{\frac{36}{9} - \frac{4x^2}{9}} = \sqrt{4 - 4/9\, x^2}$$

$$A = (2x)(2y)$$

$$= 4xy$$

$$= 4x(4 - 4/9\, x^2)^{1/2}$$

$$A' = 4(4 - 4/9\, x^2)^{1/2} + 2x(4 - 4/9\, x^2)^{-1/2}(-8/9x)$$

$$= 4(4 - 4/9\, x^2)^{1/2} - \frac{\frac{16}{9}x^2}{(4 - 4/9\, x^2)^{1/2}}$$

$$= \frac{4(4 - 4/9\, x^2) - 16/9\, x^2}{(4 - 4/9\, x^2)^{1/2}}$$

$$= \frac{16 - 16/9\, x^2 - 16/9\, x^2}{(4 - 4/9\, x^2)^{1/2}}$$

$$0 = \frac{16 - 32/9\, x^2}{(4 - 4/9\, x^2)^{1/2}}$$

$$16 = \frac{32}{9}x^2$$

$$\sqrt{\frac{16(9)}{32}} = x$$

$$\sqrt{\frac{9}{2}} = x$$

$$\frac{3}{\sqrt{2}} = x$$

$$y = \sqrt{4 - \frac{4}{9}\left(\frac{3}{\sqrt{2}}\right)^2}$$

$$= \sqrt{4 - \frac{4}{9}\frac{(9)}{2}}$$

$$= \sqrt{4 - 2}$$

$$= \sqrt{2}$$

$$A = 4\left(\frac{3}{\sqrt{2}}\right)(\sqrt{2})$$

$$= 12$$

★ error → |actual − theoretical|

percentage error → $\dfrac{|\text{actual} - \text{theoretical}|}{\text{actual}}$

7) Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate values of $f(x)$. Which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5? 1998 BC 92 Calc

(A) 2.4 (B) 2.5 (C) 2.6 (D) 2.7 (E) 2.8

$L(x) = f(a) - f'(a)(x-a)$

$f'(x) = 2x - 2$

$f'(2) = 2$

$f(2) = 4 - 4 + 3 = 3$

$L(x) = 3 + 2(x-2)$

$= 3 + 2x - 4 = 2x - 1$

$0.5 > x^2 - 2x + 3 - (2x - 1)$

$0.5 > x^2 - 4x + 4$

$0.5 > (x-2)^2$

$\pm\sqrt{0.5} > x - 2$

$-\sqrt{0.5} < x - 2 < \sqrt{0.5}$

$2 - \sqrt{0.5} < x < 2 + \sqrt{0.5}$

$1.293 < x < 2.707$

22) The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

2003 BC 78 Calc

(A) $0.04\pi\ m^2/\text{sec}$
(B) $0.4\pi\ m^2/\text{sec}$
(C) $4\pi\ m^2/\text{sec}$
(D) $20\pi\ m^2/\text{sec}$
(E) $100\pi\ m^2/\text{sec}$

$\frac{dr}{dt} = 0.2\ m/sec$

$C = 20\pi = 2\pi r$

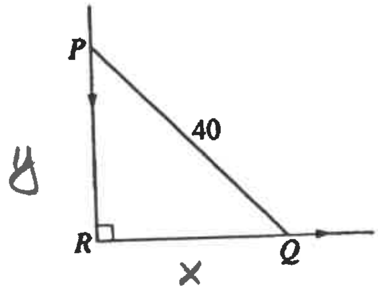
$r = 10$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(10)(0.2)$

$= 4\pi\ m^2/sec$



1993 BC 34 calc

3) In the figure above, PQ represents a 40-foot ladder with end P against a vertical wall and end Q on level ground. If the ladder is slipping down the wall, what is the distance RQ at the instant when Q is moving along the ground $\frac{3}{4}$ as fast as P is moving down the wall?

(A) $\frac{6}{5}\sqrt{10}$ (B) $\frac{8}{5}\sqrt{10}$ (C) $\frac{80}{\sqrt{7}}$ (D) 24 (E) 32

$x = ?$

$\frac{dx}{dt} = \frac{3}{4}\frac{dy}{dt}$

$x^2 + y^2 = 40^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2x\left(\frac{3}{4}\frac{dy}{dt}\right) + 2y\frac{dy}{dt} = 0$

$y = -\frac{3}{4}x$

$\frac{25}{16}x^2 = 1600$

$x = \sqrt{1600 \cdot 16/25}$

$= 32$

$x^2 + y^2 = 40^2$

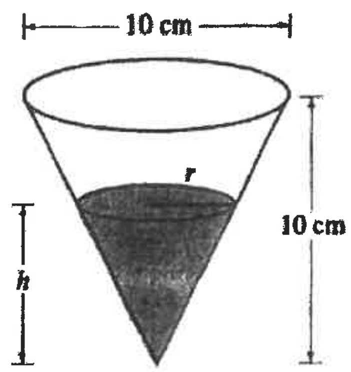
$x^2 + \left(-\frac{3}{4}x\right)^2 = 40^2$

$x^2 + \frac{9}{16}x^2 = 40^2$

$\frac{25}{16}x^2 = 40^2$

2002 AB 5

No Calc



$D = 10cm$

$r = 5cm$

$r = \frac{1}{2}h$

24 A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

$\frac{dh}{dt} = -3/10$ (Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

* $y = kx$

$\frac{dV}{dt} = k \text{ area}$

(a) $V = ?$ $h = 5cm$

$r = \frac{1}{2}h$

$r = 5/2 \text{ cm}$

$V(5) = \frac{1}{3}\pi (5/2)^2 (5)$

$= \frac{125\pi}{12} \text{ cm}^3$

(b) $\frac{dV}{dt} = ?$ $h = 5cm$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (1/2 h)^2 h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$= \frac{1}{4}\pi (5)^2 (-3/10)$

$= -\frac{15}{8} \text{ cm}^3/\text{hr}$

(c) $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$= \frac{1}{4}\pi h^2 (-3/10)$

$= -\frac{3}{40}\pi h^2$

$h = 2r$

$= -\frac{3}{40}\pi (2r)^2$

$= -\frac{3}{10}\pi r^2$

$= -\frac{3}{10} \text{ area}$

constant of proportionality is $-3/10$