BC Calculus
Chapter 6 and 7 Review

No calculator unless the problem says "calc".

1. List the formulas for:
   a. RRAM:
   b. LRAM:
   c. Trapezoidal Sum:
   d. Exponential Growth or Decay:
   e. Logistic Growth:
2. What is the formula for integration by parts?
3. How do we integrate by Partial Fractions?

4. What is the order of integration techniques one should try when approached with an integration problem?

5) $\frac{d}{dx}\left(\int_0^{x^3} \ln(t^2+1)\,dt\right) =$ 2003 BC 27

(A) $\frac{2x^3}{x^6+1}$
(B) $\frac{3x^2}{x^6+1}$
(C) $\ln(x^6+1)$
(D) $2x^3\ln(x^6+1)$
(E) $3x^2\ln(x^6+1)$

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6) Given $f(x) = \begin{cases} x+1 & \text{for } x<0, \\ \cos \pi x & \text{for } x \geq 0, \end{cases}$ $\int_{-1}^{1} f(x)\,dx =$ 1973 BC 41

(A) $\frac{1}{2}+\frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}-\frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2}+\pi$

7) Which of the following is equal to $\int_0^{\pi} \sin x \, dx$? 1993 BC 33 CALC

(A) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$ (B) $\int_0^{\pi} \cos x \, dx$ (C) $\int_{-\pi}^{0} \sin x \, dx$

(D) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx$ (E) $\int_{\pi}^{2\pi} \sin x \, dx$

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8) If $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \le x \le 2, \\ f(x) = x^2 & \text{elsewhere}, \end{cases}$ then $\int_{-1}^{3} f(x)\,dx$ is a number between 1969 BC 41

(A) 0 and 8 (B) 8 and 16 (C) 16 and 24 (D) 24 and 32 (E) 32 and 40

9) $\int_1^e \frac{x^2+1}{x} dx =$ 2008 BC 13

(A) $\frac{e^2-1}{2}$ (B) $\frac{e^2+1}{2}$ (C) $\frac{e^2+2}{2}$ (D) $\frac{e^2-1}{e^2}$ (E) $\frac{2e^2-8e+6}{3e}$

10) $\int_0^1 \sqrt{x}\,(x+1)\,dx =$

1997 BC 1

(A) 0 (B) 1 (C) $\frac{16}{15}$ (D) $\frac{7}{5}$ (E) 2

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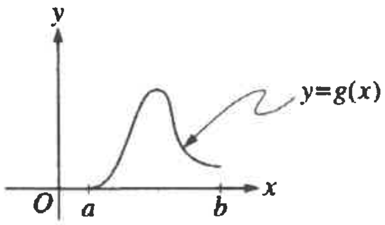
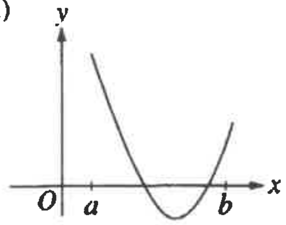
11)



1998 BC 88 Calc

Let $g(x)=\int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a,b]$. Which of the following could be the graph of f on $[a,b]$?

(A)



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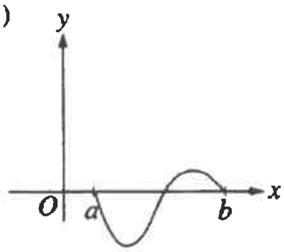

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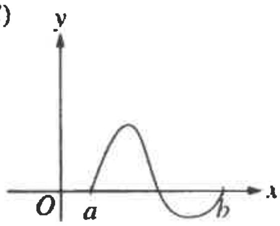

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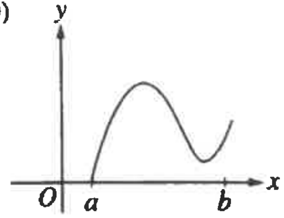

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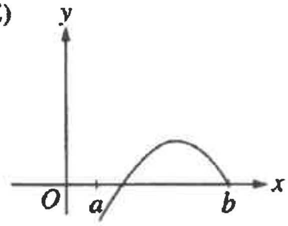

12) What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \le t \le 2$? 1969 BC 33

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

13) The average (mean) value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is 1973 BC 34

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

14) $\int_0^2 \sqrt{4-x^2}\,dx =$ 1988 BC 31

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) π (D) 2π (E) 4π

5) $\int_2^3 \frac{3}{(x-1)(x+2)}\,dx =$ 1988 BC 17

(A) $-\frac{33}{20}$ (B) $-\frac{9}{20}$ (C) $\ln\left(\frac{5}{2}\right)$ (D) $\ln\left(\frac{8}{5}\right)$ (E) $\ln\left(\frac{2}{5}\right)$

16) $\int x \sec^2 x \, dx =$ 1993 BC 29

(A) $x \tan x + C$

(B) $\frac{x^2}{2} \tan x + C$

(C) $\sec^2 x + 2\sec^2 x \tan x + C$

(D) $x \tan x - \ln|\cos x| + C$

(E) $x \tan x + \ln|\cos x| + C$

17) $\int x \cos x \, dx =$ 1998 BC 15

(A) $x \sin x - \cos x + C$

(B) $x \sin x + \cos x + C$

(C) $-x \sin x + \cos x + C$

(D) $x \sin x + C$

(E) $\frac{1}{2} x^2 \sin x + C$

18) $\int \frac{7x}{(2x-3)(x+2)}dx =$

2008 BC 19

(A) $\frac{3}{2}\ln|2x-3|+2\ln|x+2|+C$

(B) $3\ln|2x-3|+2\ln|x+2|+C$

(C) $3\ln|2x-3|-2\ln|x+2|+C$

(D) $-\frac{6}{(2x-3)^2}-\frac{2}{(x+2)^2}+C$

(E) $-\frac{3}{(2x-3)^2}-\frac{2}{(x+2)^2}+C$

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19) $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

1973 BC 21

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

20)

x	0	1
$f(x)$	2	4
$f'(x)$	6	–3
$g(x)$	–4	3
$g'(x)$	2	–1

2008 BC 22

The table above gives values of f, f', g and g' for selected values of x. If $\int_0^1 f'(x)g(x)\,dx = 5$, then $\int_0^1 f(x)g'(x)\,dx =$

(A) –14 (B) –13 (C) –2 (D) 7 (E) 15

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21) $\int_1^2 \frac{x+1}{x^2+2x}\,dx =$

1985 BC 3

(A) $\ln 8 - \ln 3$ (B) $\frac{\ln 8 - \ln 3}{2}$ (C) $\ln 8$ (D) $\frac{3\ln 2}{2}$ (E) $\frac{3\ln 2 + 2}{2}$

22) $\int_0^{\pi/4} \tan^2 x\,dx =$ 1973 BC 25

(A) $\frac{\pi}{4}-1$ (B) $1-\frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\frac{\pi}{4}+1$

23) $\int (3x+1)^5\,dx =$

2003 BC 3

(A) $\frac{(3x+1)^6}{18}+C$

(B) $\frac{(3x+1)^6}{6}+C$

(C) $\frac{(3x+1)^6}{2}+C$

(D) $\frac{(\frac{3x^2}{2}+x)^6}{2}+C$

(E) $(\frac{3x^2}{2}+x)^5+C$

24) $\int x^2 \cos\left(x^3\right) dx =$

2003 BC 8

(A) $-\frac{1}{3}\sin\left(x^3\right)+C$

(B) $\frac{1}{3}\sin\left(x^3\right)+C$

(C) $-\frac{x^3}{3}\sin\left(x^3\right)+C$

(D) $\frac{x^3}{3}\sin\left(x^3\right)+C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right)+C$

25) $\int xe^{x^2} dx =$

2008 BC 2

(A) $\frac{1}{2}e^{x^2}+C$ (B) $e^{x^2}+C$ (C) $xe^{x^2}+C$ (D) $\frac{1}{2}e^{2x}+C$ (E) $e^{2x}+C$

26) $\int_0^8 \frac{dx}{\sqrt{1+x}} =$ 1969 BC 4

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

x	2	5	10	14
$f(x)$	12	28	34	30

27)

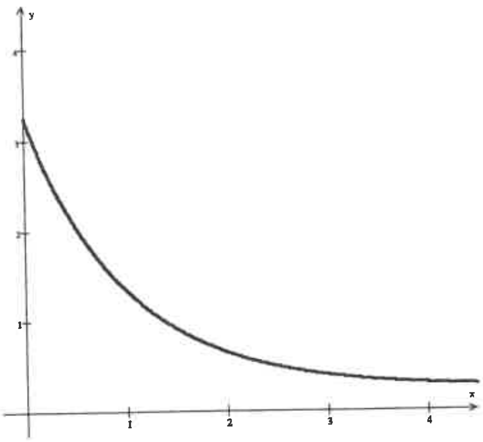
2003 BC 25

The function f is continuous on the closed interval [2,14] and has values as shown in the table above. Using the subintervals [2,5], [5,10], and [10,14], what is the approximation of $\int_2^{14} f(x)dx$ found by using a right Riemann sum?

(A) 296
(B) 312
(C) 343
(D) 374
(E) 390

28) If a trapezoidal sum overapproximates $\int_0^4 f(x)dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?

2003 BC 85
Calc



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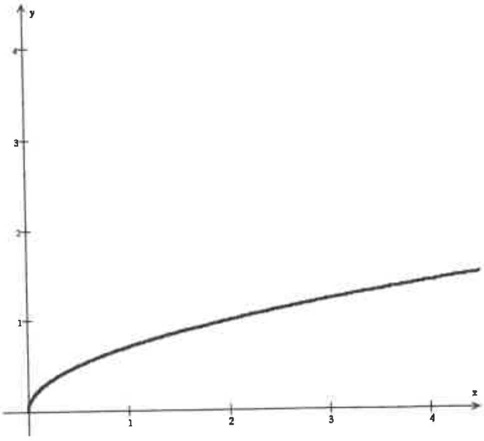

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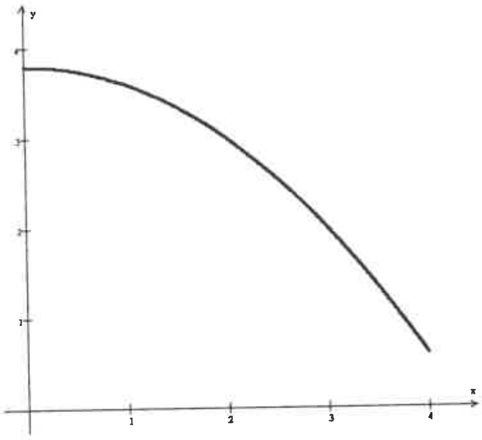

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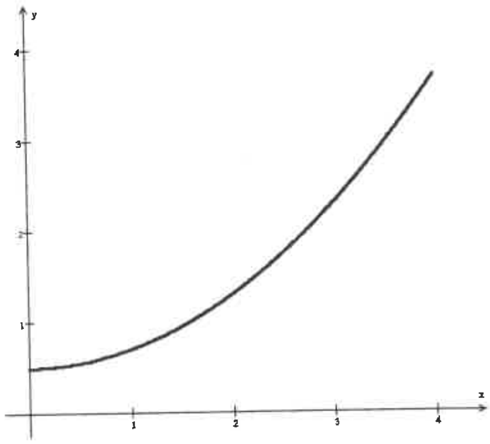

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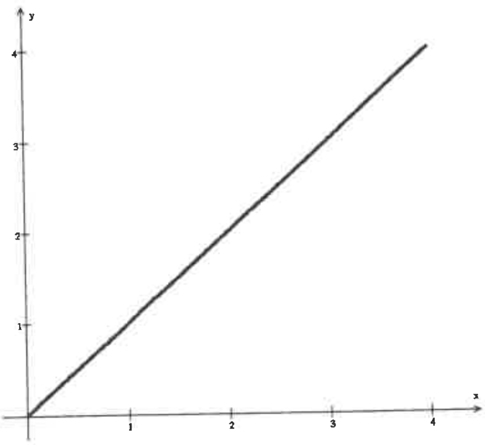


(E)

29) If three equal subdivisions of $[-4, 2]$ are used, what is the trapezoidal approximation of $\int_{-4}^{2} \frac{e^{-x}}{2} dx$? 1988 BC 18

(A) $e^2 + e^0 + e^{-2}$

(B) $e^4 + e^2 + e^0$

(C) $e^4 + 2e^2 + 2e^0 + e^{-2}$

(D) $\frac{1}{2}\left(e^4 + e^2 + e^0 + e^{-2}\right)$

(E) $\frac{1}{2}\left(e^4 + 2e^2 + 2e^0 + e^{-2}\right)$

30) Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5? 2003 BC 5

(A) 3

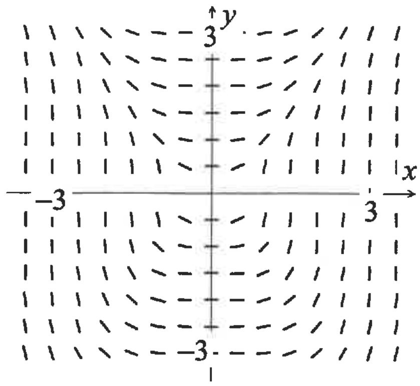
(B) 5

(C) 6

(D) 10

(E) 12

31)



2003 BC 14

Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx}=\frac{x}{y}$

(B) $\frac{dy}{dx}=\frac{x^2}{y^2}$

(C) $\frac{dy}{dx}=\frac{x^3}{y}$

(D) $\frac{dy}{dx}=\frac{x^2}{y}$

(E) $\frac{dy}{dx}=\frac{x^3}{y^2}$

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32) The general solution for the equation $\frac{dy}{dx}+y=xe^{-x}$ is

1985 BC 37

(A) $y=\frac{x^2}{2}e^{-x}+Ce^{-x}$

(B) $y=\frac{x^2}{2}e^{-x}+e^{-x}+C$

(C) $y=-e^{-x}+\frac{C}{1+x}$

(D) $y=xe^{-x}+Ce^{-x}$

(E) $y=C_1e^{x}+C_2xe^{-x}$

33) If $\frac{dy}{dx} = x^2 y$, then y could be 1993 BC 13 Calc

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$

34) If $\frac{dy}{dx} = (1 + \ln x)\, y$ and if $y = 1$ when $x = 1$, then $y =$ 1997 BC 83 Calc

(A) $e^{\frac{x^2-1}{x^2}}$

(B) $1 + \ln x$

(C) $\ln x$

(D) $e^{2x + x\ln x - 2}$

(E) $e^{x\ln x}$

35) The number of bacteria in a culture is growing at a rate of $3,000e^{2t/5}$ per unit of time t. At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$. 1973 BC 17

(A) $1,200e^2$ (B) $3,000e^2$ (C) $7,500e^2$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

36) The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship? 2003 BC 12

(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

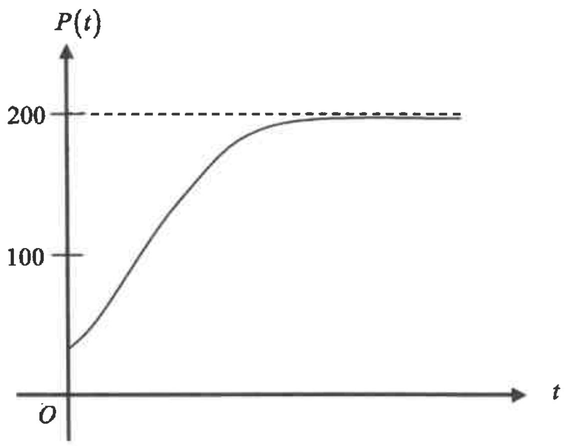
(E) $\frac{dV}{dt} = k\sqrt{V}$

37) The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M(1 - \frac{M}{200})$, where t is the time in years and $M(0) = 50$. What is $\lim_{t \to \infty} M(t)$?

2003 BC
21

(A) 50
(B) 200
(C) 500
(D) 1000
(E) 2000

38)



Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

2008 BC 24

(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$

(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$

(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$

(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$

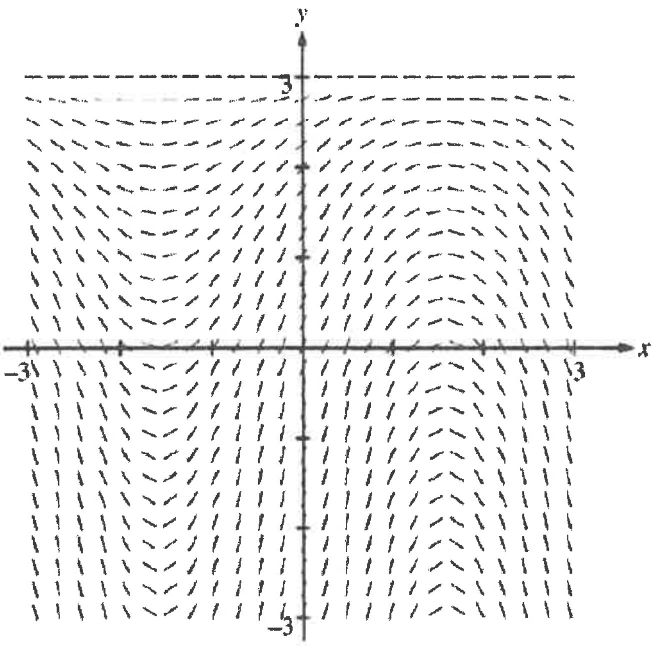
(E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$

Bonus

2014: AB-6; No Calculator

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.